10 Constructions

EXERCISE 10.1

Choose the correct answer from the given four options:

Q1. To divide a line segment AB in the ratio 5 : 7, first a ray AX is drawn so that ∠BAX is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

(*a*) 8 (*b*) 10 (*c*) 11 (*d*) 12

Sol. (*d*): Minimum number of the points marked = 5 + 7 = 12 verifies option (*d*).

Q2. To divide a line segment AB in ratio 4 : 7, a ray AX is drawn first such that ∠BAX is an acute angle and then points A_1, A_2, A_3, ... are located at equal distances on the ray AX and the point B is joined to

(*a*) A_{12} (*b*) A_{11} (*c*) A_{10} (*d*) A_9

Sol. (*b*): We have to divide the constructed line into 7 + 4 = 11 equal parts and 11th part will be joined to B. Verifies the option (*b*).

Q3. To divide a line segment AB in the ratio 5 : 6, draw a ray AX such that ∠BAX is an acute angle, then draw a ray BY parallel to AX, and the points, A_1, A_2, A_3 ... and B_1, B_2, B_3 ... are located at. Equal distances on ray AX and BY, respectively. Then the points joined are

(*a*) A_5 and B_6 (*b*) A_6 and B_5 (*c*) A_4 and B_5 (*d*) A_5 and B_4

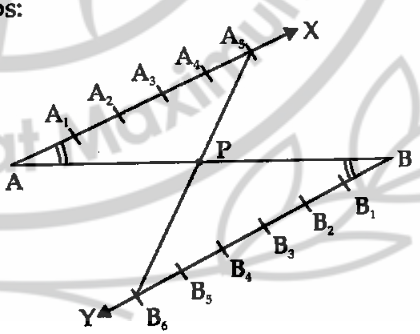
Sol. (*a*): In the figure, segment AB of given length is divided into 2 parts of ratio 5 : 6 in following steps:

(*i*) Draw a line-segment AB of given length.

(*ii*) Draw an acute angle BAX as shown in figure either up side or down side.

(*iii*) Draw angle ∠ABY = ∠BAX on other side of AX, *i.e.*, down side.

(*iv*) Divide AX into 5 equal parts by using compass.



(*v*) Divide BX into same distance in 6 equal parts as AX was divided.

(*vi*) Now, join A_5 and B_6 which meet AB at P. P divides AB in ratio AP : PB = 5 : 6.

Q4. To construct a triangle similar to a given ΔABC with its sides $\frac{3}{7}$ of the corresponding sides of ΔABC, first draw a ray BX such that $\angle CBX$ is an acute angle and X lies on the opposite side of A with respect to BC. Then locate points $B_1, B_2, B_3, \ldots$ on BX at equal distances and next step is to join

(*a*) B_{10} to C (*b*) B_3 to C (*c*) B_7 to C (*d*) B_4 to C

Sol. (*c*): Here, ratio is $\frac{3}{7} < 1$ so resultant figure will be smaller than original so, last 7th part is to be joined to C, so that parallel line from third part of BX meet on BC without producing. So, verifies the option (*c*).

Q5. To construct a triangle similar to a given ΔABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC draw a ray BX such that $\angle CBX$ is an acute angle and X is on the opposite side of A with respect to BC. The minimum number of points to be located at equal distances on the ray BX

(*a*) 5 (*b*) 8 (*c*) 13 (*d*) 3

Sol. (*b*): To construct a triangle similar to a given triangle ABC with its sides $\frac{8}{5}$ of the corresponding sides of ΔABC, the minimum number of parts in which BX is divided in 8 equal parts. Verifies the option (*b*).

Q6. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at end points of those two radii of the circle, the angle between them should be

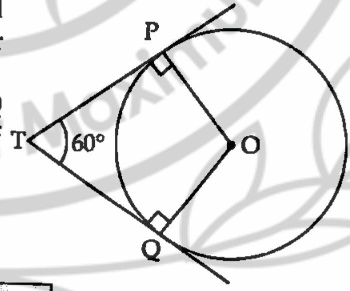
(*a*) 135° (*b*) 90° (*c*) 60° (*d*) 120°

Sol. (*d*): We know that tangent and radius at contact point are perpendicular to each other.

So, $\angle P$ and $\angle Q$ in quadrilateral TPOQ formed by tangents and radii will be of 90° each. So, the sum of $\angle T + \angle O = 180°$ as $T = 60°$ (Given)

$\therefore \angle O = 180° - 60° = 120°$

Verifies the option (*d*).



EXERCISE 10.2

Write True or False and give reason for your answer in each of the following:

Q1. By geometrical construction, it is possible to divide a line segment in ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Sol. True: On multiplying or dividing a given ratio by a real number, the ratio remains same.

On multiplying the given ratio by $\sqrt{3}$ we get $\sqrt{3}.\sqrt{3} : \frac{1}{\sqrt{3}}.\sqrt{3}$ or $3 : 1$

Hence, the given ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$ is possible to divide a line in ratio 3 : 1 in place of $\sqrt{3} : \frac{1}{\sqrt{3}}$.

Q2. To construct a triangle similar to a given ΔABC with its sides $\frac{7}{3}$ of the corresponding sides of ΔABC, draw a ray BX making acute angle with BC and X lies on the opposite side of A with respect to BC. The points $B_1, B_2, \ldots B_7$ are located at equal distances on BX, B_3 is joined to C and then a line segment B_6C' is drawn parallel to B_3C where C' lies on BC produced. Finally, the line segment A'C' is drawn parallel to AC.

Sol. False: Given ratio is $\frac{7}{3} > 1$ so, the resulting triangle will be larger than given as $B_7C' \parallel B_3C$ and BX is equally divided into 7 parts as $(7 > 3)$.

Construction: (*i*) Draw given triangle with given specifications.

(*ii*) Draw an acute angle CBX.

(*iii*) Divide BX into 7 equal parts and mark them $B_1, B_2, B_3, \ldots B_7$.

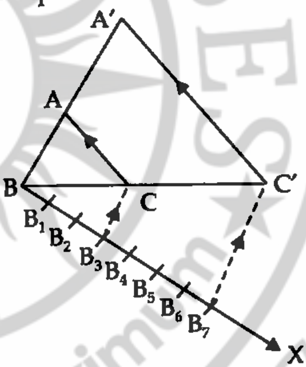
(*iv*) Produce BC and BA as shown in figure.

(*v*) Join B_3C.

(*vi*) Draw $B_7C' \parallel B_3C$, C' is on BC produced.

(*vii*) Draw $C'A' \parallel AC$. A' on BA produced $\Delta A'BC'$ is required triangle i.e., $\frac{\Delta A'BC'}{\Delta ABC} = \frac{3}{7}$.

Here, $B_7C' \parallel B_3C$. But in Question $B_6C' \parallel B_3C$, which is false.



Q3. A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm situated at a distance of 3 cm from the centre.

Sol. False: Any tangent on a circle can be drawn only if the distance of point to draw tangent is equal to or more than radius of circle. Here, radius of circle is 3.5 cm and point is at 3 cm from centre which is inside the circle. So, no tangent can be drawn if point is inside the circle.

Q4. A pair of tangents can be constructed to a circle inclined at an angle of 170°.

Sol. True: A pair of tangents can be constructed if the angle between the tangents is between zero and less than 180°. Because the sum of angles between tangents and radii on tangent are supplementary.

So, a pair of tangents can be constructed to circle inclined at an angle of 170°.

EXERCISE 10.3

Q1. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3 : 5.

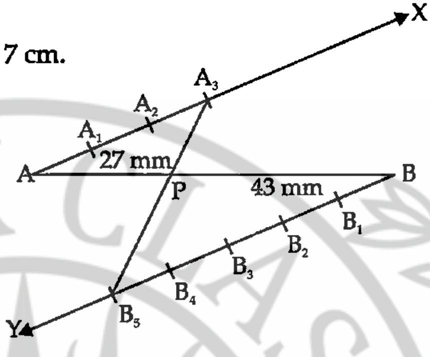
Sol. Steps of construction:

(*i*) Draw a line-segment AB = 7 cm.

(*ii*) Draw AX || BY such that ∠A and ∠B are acute angles.

(*iii*) Divide AX and BY in 3 and 5 parts equally by compass and mark $A_1, A_2, A_3, B_1, B_2, B_3, B_4$ and B_5 respectively.

(*iv*) Join A_3B_5 which intersect AB at P and divides AP : PB = 3 : 5.



Hence, P is the required point on AB which divide it in 3 : 5.

Verification (Justification): In ΔAA_3P and ΔBB_5P

	AX || BY	[By construction]
	∠A = ∠B	[Alt. angles]
	$\angle A_3PA = \angle B_5PB$	[Vertically opp. angles]
∴	$\Delta AA_3P \sim \Delta BB_5P$	[By AA criterion of similarly]

$$\Rightarrow \quad \frac{AA_3}{BB_5} = \frac{AP}{BP}$$

[Let each equal part = x cm
∴ $AA_1 = A_1A_2 = B_1B_2 ... = x$]

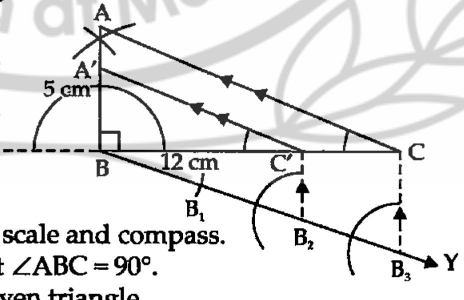
$$\Rightarrow \quad \frac{3x}{5x} = \frac{AP}{BP}$$

$$\Rightarrow \quad AP : BP = 3 : 5.$$

Hence, verified.

Q2. Draw a right angled ΔABC in which BC = 12 cm, AB = 5 cm, and ∠B = 90°. Construct a triangle similar to it and of scale factor $\frac{2}{3}$. Is the new triangle also a right triangle?

Sol. Here, scale factor or ratio factor is $\frac{2}{3} < 1$, so triangle to be constructed will be smaller than given ΔABC.



Steps of construction:

(*i*) Draw BC = 12 cm.

(*ii*) Draw ∠CBA = 90° with scale and compass.

(*iii*) Cut BA = 5 cm such that ∠ABC = 90°.

(*iv*) Join AC. ΔABC is the given triangle.

(*v*) Draw an acute ∠CBY such that A and Y are in opposite direction with respect to BC.

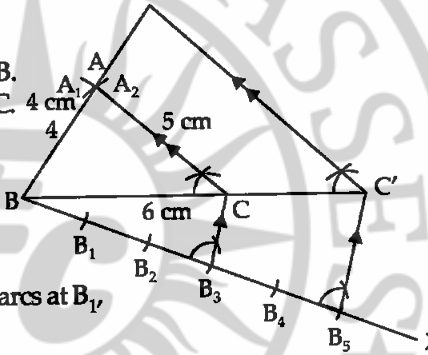
(vi) Divide BY in 3 equal segments by marking arc at same distance at B_1, B_2 and B_3.
(vii) Join B_3C.
(viii) Draw $B_2C' \parallel B_3C$ by making equal alternate angles at B_2 and B_3.
(ix) From point C', draw $C'A' \parallel CA$ by making equal alternate angles at C and C'.

$\Delta A'BC'$ is the required triangle of scale factor $\frac{2}{3}$. This triangle is also a right triangle.

Q3. Draw a ΔABC in which BC = 6 cm, CA = 5 cm and AB = 4 cm. Construct a triangle similar to it and of scale factor $\frac{5}{3}$.

Sol. Here, scale factor is $\frac{5}{3} > 1$, so the resulting figure will be larger.

Steps of construction:



(i) Draw BC = 6 cm.
(ii) Draw arc BA_1 = 4 cm from B.
(iii) Draw arc CA_2 = 5 cm from C.
(iv) Arc CA_2 and BA_1 intersect at A.
(v) Join AB and AC.
(vi) Draw acute angle CBX below BC.
(vii) Cut BX into equal parts by arcs at B_1, B_2, B_3, B_4 and B_5.
(viii) Join B_3C.
(ix) Draw $B_5C' \parallel B_3C$ by making alternate angles. C' is on BC produced.
(x) Draw $C'A' \parallel CA$ which meet BA produced at A'. Now, $\Delta A'BC'$ is the required triangle.

Justification:

$\Delta BCB_3 \sim \Delta BC'B_5$ [By AA criterion of similarity]

$\therefore \quad \frac{BB_3}{BB_5} = \frac{BC}{BC'}$ $[BB_1 = B_1B_2 = ... x$ $\therefore BB_3 = 3x$ and $BB_5 = 5x]$

$\Rightarrow \quad \frac{3x}{5x} = \frac{BC}{BC'}$

$\Delta ABC \sim \Delta A'BC'$ [By AA criterion of similarity]

$\Rightarrow \quad \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC}{BC'}$

Q4. Construct a pair of tangents to a circle of radius 4 cm from a point which is at a distance of 6 cm from the centre of circle.

Sol. The distance of point from which tangents to be drawn should be more than radius so that tangents can be drawn.

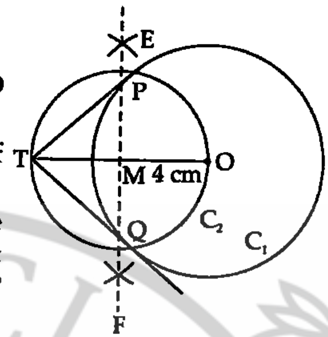
Steps of construction:

(*i*) Draw a line-segment OT = 6 cm.

(*ii*) Draw a circle of radius 4 cm taking O as centre.

(*iii*) Draw perpendicular bisector EF of OT which meets OT at M.

(*iv*) Taking MT as radius and M as centre draw a circle C_2 which intersect C_1 at P and Q. Join TP and TQ. Then, TP and TQ are the required tangents.



EXERCISE 10.4

Q1. Two line-segments AB and AC include an angle of 60°, where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC respectively such that AP = $\frac{3}{4}$ AB and AQ = $\frac{1}{4}$ AC. Join P and Q and measure the length PQ.

Sol. (*i*) Draw ∠BAC = 60° such that AB = 5 cm and AC = 7 cm.

(*ii*) Draw acute angle CAX and mark X_1, X_2, X_3 and X_4 equally spaced.

(*iii*) Join X_4C.

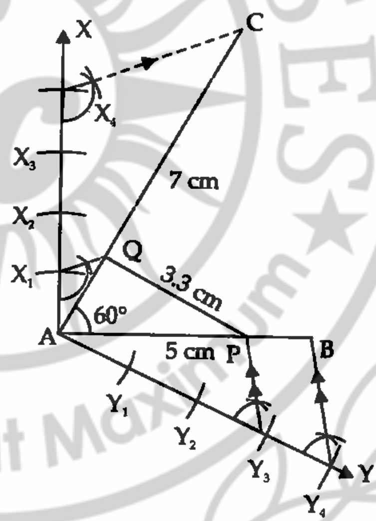
(*iv*) Draw X_1Q | | X_4C.

(*v*) Similarly, draw ∠BAY and divide AY in 4 equal parts *i.e.*, Y_1, Y_2, Y_3 and Y_4.

(*vi*) Join Y_4B and draw Y_3P | | Y_4B.

(*vii*) Join PQ and measure it.

(*viii*) PQ is equal to 3.3 cm.



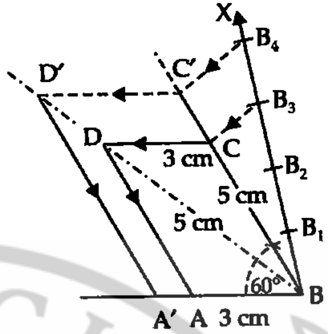
Q2. Draw a parallelogram ABCD in which BC = 5 cm, AB = 3 cm and ∠ABC = 60°. Divide it into triangles BCD and ΔABD, by diagonal BD. Construct the triangle BD′C′ similar to ΔBDC with scale factor $\frac{4}{3}$. Draw the line segment D′A′ parallel to DA, where A′ lies on extended side BA. Is A′BC′D′ a parallelogram?

Sol. Steps of construction:

(*i*) Draw a line segment AB = 3 cm.

(*ii*) Make ∠ABC = 60° such that BC = 5 cm.

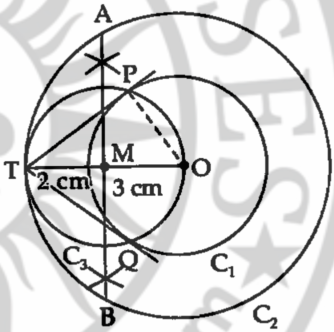
(*iii*) Draw CD ∥ AB and AD ∥ BC, ▭ABCD is the required parallelogram.



(iv) Join diagonal BD and produce it.
(v) Make acute angle CBX on opposite of D with respect to BC.
(vi) Mark (equi spaced) B_1, B_2, B_3, B_4 by compass.
(vii) Join B_3C and draw $B_3C \parallel B_4C'$ on BC produced.
(viii) Again, draw $C'D' \parallel CD$, where D′ is on BD produced.
(ix) Now, draw $D'A' \parallel DA$ where A′ is on BA produced. Parallelogram A′BC′D′ is similar to parallelogram ABCD with scale factor $\frac{4}{3}$.

Q3. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

Sol. Steps of construction:



(i) Draw two concentric circles C_1, C_2 of radii 3 cm and 5 cm respectively taking 'O' as centre.
(ii) Draw perpendicular bisector AB of OT. T is any point on C_2.
(iii) Draw circle C_3 taking radius TM = OM and M as centre.
(iv) Circle C_3 intersect the circle C_1 at P and Q. Join TP and TQ. These are the required tangents. TP = TQ = 4.1 cm by measuring.

Mathematically length of tangent: Join OP. OP and TP are radius and tangent respectively at contact point P. So, $\angle TPO = 90°$.

By Pythagoras theorem in ΔTPO,

$$PT^2 = OT^2 - OP^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\Rightarrow \quad PT = 4 \text{ cm}$$

Difference in measurement and by mathematical calculation

$$PT = 4.1 \text{ cm} - 4 \text{ cm} = 0.1 \text{ cm.}$$

Q4. Draw an isosceles ΔABC in which AB = AC = 6 cm and BC = 5 cm. Construct a triangle PQR similar to ΔABC in which PQ = 8 cm. Also justify the construction.

Sol. We have to draw

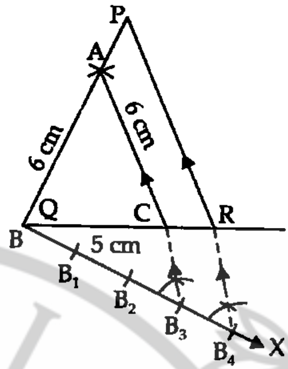
$\Delta PQR \sim \Delta ABC$

$PQ = 8$ cm

$\therefore \quad \frac{PQ}{AB} = \frac{8}{6} = \frac{4}{3} \quad (\because AB = 6 \text{ cm})$

So, $\quad PQ = QR = 8$ cm

So, we have to draw $\Delta PQR \sim \Delta ABC$ with scale factor $\frac{4}{3} > 1$ resulting ΔPQR will be larger than ΔABC.



Steps of Construction:

(*i*) Draw BC = 5 cm

(*ii*) Draw two arcs of 6 cm each from B and C in same direction let it be upside.

(*iii*) Join AB and AC.

(*iv*) Draw acute $\angle CBX$ and mark B, B_1, B_2, B_3, B_4 with compass.

(*v*) Join B_3C and draw $B_4R \parallel B_3C$, R is on BC produced.

(*vi*) Again, draw $RP \parallel CA$. P is on BA produced.

Therefore, $\Delta PQR \sim \Delta ABC$ with PQ = PR = 8 cm. It's scale factor is $\frac{4}{3}$.

Q5. Draw a ΔABC in which AB = 5 cm, BC = 6 cm and $\angle ABC = 60°$. Construct a triangle similar to ΔABC with scale factor $\frac{5}{7}$. Justify the construction.

Sol. Scale factor $\frac{5}{7} < 1$, so the resulting Δ will be smaller than ΔABC.

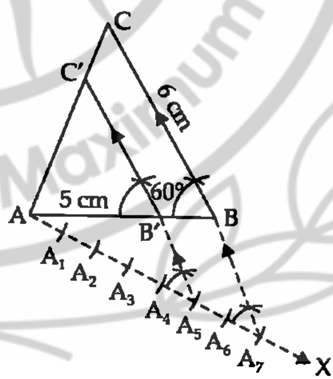
Steps of construction:

(*i*) Draw AB = 5 cm.

(*ii*) Draw $\angle ABC = 60°$, cut BC = 6 cm and join AC.

(*iii*) Draw acute $\angle BAX$ and mark it equispaced marks $A_1, A_2, ..., A_7$ as shown in figure.

(*iv*) Join A_7B and draw $A_5B' \parallel A_7B$. B′ is on segment AB.



Draw $B'C' \parallel BC$, point C′ is on AC.

$\Delta AB'C' \sim \Delta ABC$ with scale factor $\frac{5}{7}$.

Justification: In $\Delta AA_5B'$ and AA_7B,

$A_7B \parallel A_5B'$

$\therefore \quad \angle A_5 = \angle A_7 \quad$ [Corresponding $\angle$s]

$\angle BAA_5 = \angle BAA_7 \quad$ [Common]

$\therefore \quad \Delta AA_5B' \sim \Delta AA_7B$ [By AA criterion of similarity]

$$\Rightarrow \quad \frac{AB'}{AB} = \frac{AA_5}{AA_7} = \frac{5x}{7x} = \frac{5}{7} \quad ...(i)$$

where $x = AA_1 = A_1A_2 = ...A_6A_7$

Similarly, $\Delta AB'C' \sim \Delta ABC$ [By AA criterion of similarity]

$$\Rightarrow \quad \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC}$$

$$\Rightarrow \quad \frac{5}{7} = \frac{AC'}{AC} = \frac{B'C'}{BC}$$

Hence, $\Delta AB'C' \sim \Delta ABC$ with scale factor $\frac{5}{7}$.

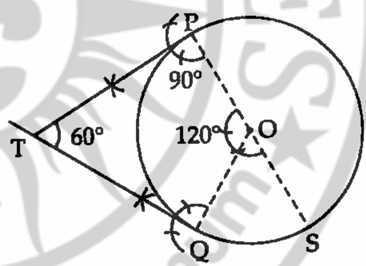
Q6. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is 60°. Also, justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.

Sol. Angle between tangents is 60°. So angles between their radii is $180° - 60° = 120°$.

As the angles between tangents and their corresponding radii are supplementary.

Steps of construction:

(i) Draw a circle of radius 4 cm.
(ii) Draw any diameter POS.
(iii) Draw OQ making ∠AOC = 120°.
(iv) Draw tangent at P by drawing ∠OPT = 90°.
(v) Similarly, draw ∠OQT equal to 90° to draw tangent.
(vi) Both PT, QT tangents intersect at T and make angle of 60°.



Hence, the two tangents on circle are TP and TQ inclined at 60°.

Justification: Because the radius OP and tangent PT at contact point makes angle ∠TPO = 90°.

Similarly, ∠TQO = 90°

In quadrilateral TPOQ,

$\angle T + \angle P + \angle O + \angle Q = 360°$

$\Rightarrow \angle T + 90° + 120° + 90° = 360°$ [$\because \angle O = 120°$ by construction]

$\Rightarrow \quad \angle T = 360° - 300°$

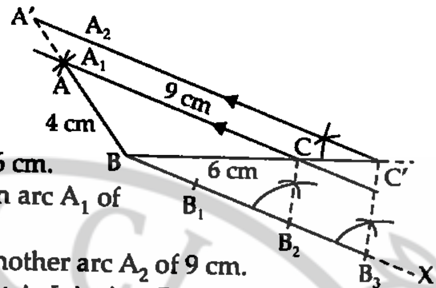
$\Rightarrow \quad \angle T = 60°$.

Hence, verified.

Q7. Draw a ΔABC in which AB = 4 cm, BC = 6 cm, and AC = 9 cm. Construct a triangle similar to ΔABC with scale factor $\frac{3}{2}$. Justify the construction. Are the two triangles congruent? Note that, all the three angles and two sides of the two triangles are equal.

Sol. Scale factor $\frac{3}{2} > 1$

So, the resulting figure will be greater than ΔABC.



Steps of construction:

(i) Draw line segment BC = 6 cm.
(ii) From B as centre, draw an arc A_1 of 6 cm.
(iii) From C as centre, draw another arc A_2 of 9 cm.
(iv) Arcs A_1 and A_2 intersect at A. Join A to B and C.
(v) Make an acute angle of $\angle CBX$ on other side of A.
(vi) Make the equispaced marks B_1, B_2, B_3 with compass.
(vii) Join B_2C and draw $B_3C' \parallel B_2C$, where C′ is on BC produced.
(viii) Draw $CA \parallel C'A'$, where A′ is on BA produced.

$\therefore \Delta A'BC' \sim \Delta ABC$ with scale factor $\frac{3}{2}$.

Justification: In $\Delta BB_3C'$ and ΔBB_2C

$\angle B = \angle B$ [Common]

$B_3C' \parallel B_2C$ [By construction]

$\therefore \angle BB_2C = \angle BB_3C'$ [Corresponding angles)

$\therefore \Delta BB_3C' \sim \Delta BB_2C$ [By AA criterion of similarity]

$\Rightarrow \frac{BC'}{CB} = \frac{BB_3}{BB_2} = \frac{3x}{2x} = \frac{3}{2}$ $[\because BB_1 = B_1B_2 = B_2B_3 = x]$

$\Rightarrow \frac{BC'}{BC} = \frac{3}{2}$

In ΔABC and $\Delta A'BC'$,

$\angle B = \angle B$ [Common]

$\because A'C' \parallel AC$

$\therefore \angle A'C'B = \angle ACB$ [Corresponding angles]

$\therefore \Delta ABC \sim \Delta A'BC'$ [By AA criterion of similarity]

$\Rightarrow \frac{A'C'}{AC} = \frac{A'B}{AB} = \frac{C'B}{BC}$

$\Rightarrow \frac{A'C'}{AC} = \frac{A'B}{AB} = \frac{3}{2}$

Hence, proved.

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