

2

Polynomials

EXERCISE 2.1

Write the correct answer in each of the following:

1. Which one of the following is a polynomial?

(a) $\frac{x^2}{2} - \frac{2}{x^2}$

(b) $\sqrt{2x} - 1$

(c) $x^2 + \frac{3x^2}{\sqrt{x}}$

(d) $\frac{x-1}{x+1}$

Sol. (a) $\frac{x^2}{2} - \frac{2}{x^2} = \frac{x^2}{2} - 2x^{-2}$

Second term is $-2x^{-2}$. Exponent of x^{-2} is -2 , which is not a whole number. So, this algebraic expression is not a polynomial.

(b) $\sqrt{2x} - 1 = \sqrt{2}x^{\frac{1}{2}} - 1$

First term is $\sqrt{2}x^{\frac{1}{2}}$. Here, the exponent of the second term, i.e., $x^{\frac{1}{2}}$ is $\frac{1}{2}$, which is not a whole number. So, this algebraic expression is not a polynomial.

(c) $x^2 + \frac{3x^2}{\sqrt{x}} = x^2 + 3x$

In this expression, we have only whole numbers as the exponents of the variable in each term. Hence, the given algebraic expression is a polynomial.

2. $\sqrt{2}$ is a polynomial of degree

(a) 2

(b) 0

(c) 1

(d) $\frac{1}{2}$

Sol. $\sqrt{2}$ is a constant polynomial. The only term here is $\sqrt{2}$ which can be written as $\sqrt{2}x^0$. So, the exponent of x is zero. Therefore, the degree of the polynomial is 0.

Hence, (b) is the correct answer.

3. Degree of the polynomial of $4x^4 + 0x^3 + 0x^2 + 5x + 7$ is

(a) 4

(b) 5

(c) 3

(d) 7

Sol. The highest power of the variable in a polynomial is called the degree of the polynomial. In this polynomial, the term with highest power of x is $4x^4$. Highest power of x is 4, so the degree of the given polynomial is 4.

4. Degree of the zero polynomial

(a) 0

(b) 1

(c) Any natural number

(d) Not defined.

Sol. Degree of the zero polynomial (0) is not defined.

Hence, (d) is the correct answer.

5. If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(a) 0

(b) 1

(c) $4\sqrt{2}$

(d) $8\sqrt{2} + 1$

Sol. We have $p(x) = x^2 - 2\sqrt{2}x + 1$

$$\begin{aligned}\therefore p(2\sqrt{2}) &= (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1 \\ &= 8 - 8 + 1 \\ &= 1\end{aligned}$$

Hence, (b) is the correct answer.

6. The value of the polynomial $5x - 4x^2 + 3$, when $x = -1$ is

(a) -6

(b) 6

(c) 2

(d) -2

Sol. Let $p(x) = 5x - 4x^2 + 3$

$$\text{Therefore, } p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$$

Hence, (a) is the correct answer.

7. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

(a) 3

(b) $2x$

(c) 0

(d) 6

Sol. We have $p(x) = x + 3$, then

$$p(-x) = -x + 3$$

$$\text{Therefore, } p(x) + p(-x) = x + 3 + (-x + 3) = x + 3 - x + 3 = 6$$

Hence, (d) is the correct answer.

8. Zero of the zero polynomial is

(a) 0

(b) 1

(c) Any real number

(d) Not defined

Sol. The zero (or degree) of the zero polynomial is undefined.

Hence, (d) is the correct answer.

9. Zero of the polynomial $p(x) = 2x + 5$ is

(a) $-\frac{2}{5}$

(b) $-\frac{5}{2}$

(c) $\frac{2}{5}$

(d) $\frac{5}{2}$

Sol. Finding a zero of $p(x)$ is the same as solving an equation $p(x) = 0$.

Now,

$$p(x) = 0 \Rightarrow 2x + 5 = 0,$$

which gives us

$$x = -\frac{5}{2}.$$

Therefore, $-\frac{5}{2}$ is the zero of the polynomial.

Hence, (b) is the correct answer.

10. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

- (a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2

Sol. We have $p(x) = 2x^2 + 7x + 4$

$$\begin{aligned} \text{(a)} \quad p(2) &= 2(2)^2 + 7(2) - 4 \\ &= 8 + 14 - 4 \\ &= 18 \neq 0 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 4 \\ &= 2 \times \frac{1}{4} + \frac{7}{2} - 4 = \frac{1}{2} + \frac{7}{2} - 4 = 4 - 4 = 0 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad p\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right) - 4 \\ &= 2 \times \frac{1}{4} - \frac{7}{2} - 4 = \frac{1}{2} - \frac{7}{2} - 4 \\ &= -3 - 4 \\ &= -7 \neq 0 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad p(-2) &= 2(-2)^2 + 7(-2) - 4 \\ &= 8 - 14 - 4 = -10 \neq 0 \end{aligned}$$

As $p\left(\frac{1}{2}\right) = 0$, we say that $\frac{1}{2}$ is a zero of the polynomial. Hence, $\frac{1}{2}$ is one of the zeroes of the polynomial $2x^2 + 7x - 4$.

Hence, (b) is the correct answer.

11. If $x^{51} + 51$ is divided by $x + 1$, the remainder is

- (a) 0 (b) 1 (c) 49 (d) 50

Sol. If $p(x)$ is divided by $x + a$, then the remainder is $p(-a)$.

Here $p(x) = x^{51} + 51$ is divided by $x + 1$, then

$$\text{Remainder} = p(-1) = (-1)^{51} + 51 = -1 + 51 = 50$$

Hence, (d) is the correct answer.

12. If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then the value of k is

- (a) -3 (b) 4 (c) 2 (d) -2

Sol. Let $p(x) = 2x^2 + kx$

If $x + 1$ is a factor of $p(x)$, then by factor theorem $p(-1) = 0$

$$\text{Now,} \quad p(-1) = 0 \Rightarrow 2(-1)^2 + k(-1) = 0$$

$$\Rightarrow \quad 2 - k = 0; \quad k = 2$$

Hence, (c) is the correct answer.

13. $x + 1$ is a factor of the polynomial

(a) $x^3 + x^2 - x + 1$

(b) $x^3 + x^2 + x + 1$

(c) $x^4 + x^3 + x^2 + 1$

(d) $x^4 + 3x^3 + 3x^2 + x + 1$

Sol. If $x + 1$ is a factor of $p(x)$, then $p(-1) = 0$

(a) Let $p(x) = x^3 + x^2 - x + 1$

$$\begin{aligned} \therefore p(-1) &= (-1)^3 + (-1)^2 - (-1) + 1 \\ &= -1 + 1 + 1 + 1 = 2 \neq 0 \end{aligned}$$

So, $x + 1$ is not a factor of $p(x)$.

(b) Let $p(x) = x^3 + x^2 + x + 1$

$$\begin{aligned} \therefore p(-1) &= (-1)^3 + (-1)^2 + (-1) + 1 \\ &= -1 + 1 - 1 + 1 = 0 \end{aligned}$$

(c) Let $p(x) = x^4 + x^3 + x^2 + 1$

$$\begin{aligned} \therefore p(-1) &= (-1)^4 + (-1)^3 + (-1)^2 + 1 \\ &= 1 - 1 + 1 + 1 = 2 \neq 0 \end{aligned}$$

(d) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$$\begin{aligned} \therefore p(-1) &= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 \\ &= 1 - 3 + 3 - 1 + 1 = 1 \neq 0 \end{aligned}$$

Hence, $x + 1$ is a factor of $x^3 + x^2 + x + 1$.

So, (b) is the correct answer.

14. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is

(a) $5 + x$

(b) $5 - x$

(c) $5x - 1$

(d) $10x$

Sol. $(25x^2 - 1) + (1 + 5x)^2 = (5x)^2 - 1^2 + (5x + 1)^2$

$$= (5x - 1)(5x + 1) + (5x + 1)^2 = (5x + 1)(5x - 1 + 5x + 1)$$

$$= (5x + 1)(10x) = 10x(5x + 1)$$

Hence, one of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is $10x$. Therefore, (d) is the correct answer.

15. The value of $249^2 - 248^2$ is

(a) 1^2

(b) 477

(c) 487

(d) 497

Sol. $(249)^2 - (248)^2 = (249 + 248)(249 - 248)$

$$= (497)(1) = 497$$

Hence, (d) is the correct answer.

16. The factorisation of $4x^2 + 8x + 3$ is

(a) $(x + 1)(x + 3)$

(b) $(2x + 1)(2x + 3)$

(c) $(2x + 2)(2x + 5)$

(d) $(2x - 1)(2x - 3)$

Sol. $4x^2 + 8x + 3 = 4x^2 + 6x + 2x + 3$

$$= 2x(2x + 3) + 1(2x + 3) = (2x + 1)(2x + 3)$$

Hence, (b) is the correct answer.

17. Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?

(a) $x^2 + y^2 + 2xy$

(b) $x^2 + y^2 - xy$

(c) xy^2

(d) $3xy$

Sol. $(x + y)^3 - (x^3 + y^3) = x^3 + y^3 + 3xy(x + y) - x^3 - y^3$

$$= 3xy(x + y)$$

So, $3xy$ is a factor of $(x + y)^3 - (x^3 + y^3)$.

Hence, (d) is the correct answer.

18. The coefficient of x in the expansion of $(x + 3)^3$ is

- (a) 1 (b) 9 (c) 18 (d) 27

Sol. Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$, we get

$$\begin{aligned}(x + 3)^3 &= x^3 + 3^3 + 3 \times x \times 3(x + 3) \\ &= x^3 + 27 + 9x^2 + 27x\end{aligned}$$

Therefore, the coefficient of x is 27.

Hence, (d) is the correct answer.

19. If $\frac{x}{y} + \frac{y}{x} = -1$, the value of $x^3 - y^3$ is

- (a) 1 (b) -1 (c) 0 (d) $\frac{1}{2}$

Sol.

$$\frac{x}{y} + \frac{y}{x} = -1 \Rightarrow \frac{x^2 + y^2}{xy} = -1$$

$$\Rightarrow x^2 + y^2 = -xy$$

Now,

$$\begin{aligned}x^3 - y^3 &= (x - y)(x^2 + y^2 + xy) \\ &= (x - y)(-xy + xy) \quad [\because x^2 + y^2 = -xy] \\ &= (x - y)(0) \\ &= 0\end{aligned}$$

Hence, (c) is the correct answer.

20. If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is

- (a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Sol.

$$49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$$

$$\begin{aligned}\Rightarrow 49x^2 - b &= (7x)^2 - \left(\frac{1}{2}\right)^2 \\ &= 49x^2 - \frac{1}{4} \quad [\because (a + b)(a - b) = a^2 - b^2]\end{aligned}$$

So, we get $b = \frac{1}{4}$.

Hence, (c) is the correct answer.

21. If $a + b + c = 0$, then the value of $a^3 + b^3 + c^3$ is equal to

- (a) 0 (b) abc (c) $3abc$ (d) $2abc$

Sol. We know that

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{As } a + b + c = 0, \text{ so, } a^3 + b^3 + c^3 - 3abc = (0)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$\text{Hence, } a^3 + b^3 + c^3 = 3abc.$$

Therefore, (c) $3abc$ is the correct answer.

EXERCISE 2.2

1. Which of the following expressions are polynomials? Justify your answer.

(i) 8

(ii) $\sqrt{3}x^2 - 2x$

(iii) $1 - \sqrt{5}x$

(iv) $\frac{1}{5x^2} + 5x + 7$

(v) $\frac{(x-2)(x-4)}{x}$

(vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

(viii) $\frac{1}{2x}$

Sol. (i) 8 is a constant polynomial.

(ii) $\sqrt{3}x^2 - 2x$

In each term of this expression, the exponent of the variable x is a whole number. Hence, it is a polynomial.

(iii) $1 - \sqrt{5}x = 1 - \sqrt{5}x^{\frac{1}{2}}$

Here, the exponent of the second term, i.e., $x^{\frac{1}{2}}$ is $\frac{1}{2}$, which is not a whole number. Hence, the given algebraic expression is not a polynomial.

(iv) $\frac{1}{5x^2} + 5x + 7 = \frac{1}{5}x^{-2} + 5x + 7$

In each term of this expression, the exponent of the variable x is a whole number. Hence, it is a polynomial.

(v) $\frac{(x-2)(x-4)}{x} = \frac{x^2 - 6x + 8}{x} = x - 6 + \frac{8}{x} = x - 6 + 8x^{-1}$

Here, the exponent of variable x in the third term, i.e., in $8x^{-1}$, is -1 , which is not a whole number. So, this algebraic expression is not a polynomial.

(vi) $\frac{1}{x+1} = (x+1)^{-1}$ which cannot be reduced to an expression in which the exponent of the variable x have only whole numbers in each of its terms. So, this algebraic expression is not a polynomial.

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

In this expression, the exponent of a in each term is a whole number, so this expression is a polynomial.

(viii) $\frac{1}{2x} = \frac{1}{2}x^{-1}$

Here, the exponent of the variable x is -1 , which is not a whole number. So, this algebraic expression is not a polynomial.

2. Write whether the following statements are true or false. Justify your answer.

(i) A binomial can have atmost two terms.

(ii) Every polynomial is a binomial.

- (iii) A binomial may have degree 5.
 (iv) Zero of a polynomial is always 0.
 (v) A polynomial cannot have more than one zero.
 (vi) The degree of the sum of two polynomials each of degree 5 is always 5.

Sol. (i) The given statement is false because binomial have exactly two terms.

(ii) A polynomial can be a monomial, binomial trinomial or can have finite number of terms. For example, $x^4 + x^3 + x^2 + 1$ is a polynomial but not binomial.

Hence, the given statement is false.

(iii) The given statement is true because a binomial is a polynomial whose degree is a whole number ≥ 1 . For example, $x^5 - 1$ is a binomial of degree 5.

(iv) The given statement is false, because zero of polynomial can be any real number.

(v) The given statement is false, because a polynomial can have any number of zeroes which depends on the degree of the polynomial.

(vi) The given statement is false. For example, consider the two polynomials $-x^5 + 3x^2 + 4$ and $x^5 + x^4 + 2x^3 + 3$. The degree of each of these polynomials is 5. Their sum is $x^4 + 2x^3 + 3x^2 + 7$. The degree of this polynomial is not 5.

EXERCISE 2.3

1. Classify the following polynomials as polynomials in one variable, two variables etc.

(i) $x^2 + x + 1$

(ii) $y^3 - 5y$

(iii) $xy + yz + zx$

(iv) $x^2 - 2xy + y^2 + 1$

Sol. (i) $x^2 + x + 1$ is a polynomial in one variable.

(ii) $y^3 - 5y$ is a polynomial in one variable.

(iii) $xy + yz + zx$ is a polynomial in three variables.

(iv) $x^2 - 2xy + y^2 + 1$ is a polynomial in two variables.

2. Determine the degree of each of the following polynomials:

(i) $2x - 1$

(ii) -10

(iii) $x^3 - 9x + 3x^5$

(iv) $y^3(1 - y^4)$

Sol. (i) Since the highest power of x is 1, the degree of the polynomial $2x - 1$ is 1.

(ii) -10 is a non-zero constant. A non-zero constant term is always regarded as having degree 0.

(iii) Since the highest power of x is 5, the degree of the polynomial $x^3 - 9x + 3x^5$ is 5.

(iv) $y^3(1 - y^4) = y^3 - y^7$

Since the highest power of y is 7, the degree of the polynomial is 7.

3. For the polynomial $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$, write

- (i) the degree of the polynomial.
- (ii) the coefficient of x^3 .
- (iii) the coefficient of x^6 .
- (iv) the constant term.

Sol. (i) We know that highest power of variable in a polynomial is the degree of the polynomial.

In the given polynomial, the term with highest of x is $-x^6$ and the exponent of x in this term is 6.

(ii) The coefficient of x^3 is $\frac{1}{5}$. (iii) The coefficient of x^6 is -1 .

(iv) The constant term is $\frac{1}{5}$.

4. Write the coefficient of x^2 in each of the following:

(i) $\frac{\pi}{6}x + x^2 - 1$

(ii) $3x - 5$

(iii) $(x-1)(3x-4)$

(iv) $(2x-5)(2x^2-3x+1)$

Sol. (i) The coefficient of x^2 in the given polynomial is 1.

(ii) The given polynomial can be written as $0 \cdot x^2 + 3x - 5$. So, the coefficient of x^2 in the given polynomial is 0.

(iii) The given polynomial can be written as:

$$\begin{aligned}(x-1)(3x-4) &= 3x^2 - 4x - 3x + 4 \\ &= 3x^2 - 7x + 4\end{aligned}$$

So, the coefficient of x^2 in the given polynomial is 3.

(iv) The given polynomial can be written as:

$$\begin{aligned}(2x-5)(2x^2-3x+1) &= 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5 \\ &= 4x^3 - 16x^2 + 17x - 5\end{aligned}$$

So, the coefficient of x^2 in the given polynomial is -16 .

5. Classify the following as a constant, linear, quadratic and cubic polynomials:

(i) $2 - x^2 + x^3$

(ii) $3x^3$

(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) $2 + x$

(vii) $y^3 - y$

(viii) $1 + x + x^2$

(ix) t^2

(x) $\sqrt{2}x - 1$

Sol. We know that

(a) a polynomial in which exponent of the variable is zero, is called a constant term.

Here, (v) 3 is a constant polynomial because $3 = 3x^0$, exponent of the variable x is 0.

(b) A polynomial of degree 1 is called a linear polynomial.

$5t - \sqrt{7}$, $2 + x$ and $\sqrt{2}x + 1$ are linear polynomials.

(c) A polynomial of degree 2 is called a quadratic polynomial.

$4 - 5y^2$, $1 + x + x^2$ and t^2 are quadratic polynomials.

(d) A polynomial of degree 3 is called a cubic polynomial.

$2 - x + x^3$, $3x^3$ and $y^3 - y$ are cubic polynomials.

6. Give an example of a polynomial, which is:

(i) monomial of degree 1.

(ii) binomial of degree 20.

(iii) trinomial of degree 2.

Sol. We know that a polynomial having only one term is called a monomial, a polynomial having only two terms is called binomial, a polynomial having only three terms is called a trinomial.

(i) $3x$ is a monomial of degree 1.

(ii) $x^{20} - 7$ is a binomial of degree 20.

(iii) $5x^2 + 3x - 1$ is a trinomial of degree 2.

7. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when $x = 3$ and also when $x = -3$.

Sol. Let $p(x) = 3x^3 - 4x^2 + 7x - 5$

$$\begin{aligned} \therefore p(3) &= 3(3)^3 - 4(3)^2 + 7(3) - 5 \\ &= 3(27) - 4(9) + 21 - 5 \\ &= 81 - 36 + 21 - 5 \\ &= 61 \end{aligned}$$

$$\begin{aligned} \text{Now, } p(-3) &= 3(-3)^3 - 4(-3)^2 + 7(-3) - 5 \\ &= 3(-27) - 4(9) - 21 - 5 \\ &= -81 - 36 - 21 - 5 \\ &= -143 \end{aligned}$$

8. If $p(x) = x^2 - 4x + 3$, evaluate $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

Sol. We have $p(x) = x^2 - 4x + 3$

$$\therefore p(2) - p(-1) + p\left(\frac{1}{2}\right) = (2^2 - 4 \times 2 + 3)$$

$$- \{(-1)^2 - 4(-1) + 3\} + \left\{ \left(\frac{1}{2}\right)^2 - 4 \times \frac{1}{2} + 3 \right\}$$

$$\begin{aligned}
 &= (4-8+3)-(1+4+3)+\left(\frac{1}{4}-2+3\right) \\
 &= -1-8+\frac{5}{4} \\
 &= -9+\frac{5}{4}=\frac{-36+5}{4}=\frac{-31}{4}
 \end{aligned}$$

9. Find $p(0)$, $p(1)$, $p(-2)$ for the following polynomials:

(i) $p(x) = 10x - 4x^2 - 3$

(ii) $p(y) = (y+2)(y-2)$

Sol. (i) We have

$$p(x) = 10x - 4x^2 - 3$$

$$\therefore p(0) = 10(0) - 4(0)^2 - 3$$

$$= 0 - 0 - 3 = -3$$

And,

$$p(1) = 10(1) - 4(1)^2 - 3$$

$$= 10 - 4 - 3 = 10 - 7 = 3$$

And,

$$p(-2) = 10(-2) - 4(-2)^2 - 3$$

$$= -20 - 4(4) - 3 = -20 - 16 - 3 = -39$$

(ii) We have $p(y) = (y+2)(y-2) = y^2 - 4$

$$\therefore p(0) = (0)^2 - 4$$

$$= 0 - 4 = -4$$

And,

$$p(1) = (1)^2 - 4$$

$$= 1 - 4 = -3$$

And,

$$p(-2) = (-2)^2 - 4$$

$$= 4 - 4 = 0$$

10. Verify whether the following are True or False.

(i) -3 is a zero of $x - 3$.

(ii) $-\frac{1}{3}$ is a zero of $3x + 1$.

(iii) $\frac{-4}{5}$ is a zero of $4 - 5y$.

(iv) 0 and 2 are the zeros of $t^2 - 2t$.

(v) -3 is a zero of $y^2 + y - 6$.

Sol. A zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$

(i) Let $p(x) = x - 3$

$$\therefore p(-3) = -3 - 3 = -6 \neq 0$$

Hence, -3 is not a zero of $x - 3$.

(ii) Let $p(x) = 3x + 1$

$$\therefore p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

Hence, $-\frac{1}{3}$ is a zero of $p(x) = 3x + 1$.

(iii) Let $p(y) = 4 - 5y$

$$\therefore p\left(-\frac{4}{5}\right) = 4 - 5\left(-\frac{4}{5}\right) = 4 + 4 = 8 \neq 0$$

Hence, $-\frac{4}{5}$ is not a zero of $4 - 5y$.

(iv) Let $p(t) = t^2 - 2t$

$$\therefore p(0) = (0)^2 - 2(0) = 0$$

$$\text{and } p(2) = (2)^2 - 2(2) = 4 - 4 = 0$$

Hence, 0 and 2 are zeroes of the polynomial $p(t) = t^2 - 2t$.

(v) Let $p(y) = y^2 + y - 6$

$$\therefore p(-3) = (-3)^2 + (-3) - 6 = 9 - 3 - 6 = 0$$

Hence, -3 is a zero of the polynomial $y^2 + y - 6$.

11. Find the zeroes of the polynomial in each of the following:

(i) $p(x) = x - 4$

(ii) $g(x) = 3 - 6x$

(iii) $q(x) = 2x - 7$

(iv) $h(y) = 2y$

Sol. (i) Solving the equation $p(x) = 0$, we get

$$x - 4 = 0, \text{ which gives us } x = 4$$

So, 4 is a zero of the polynomial $x - 4$.

(ii) Solving the equation $g(x) = 0$, we get

$$3 - 6x = 0, \text{ which gives us } x = \frac{1}{2}$$

So, $\frac{1}{2}$ is a zero of the polynomial $3 - 6x$.

(iii) Solving the equation $q(x) = 0$, we get

$$2x - 7 = 0, \text{ which gives us } x = \frac{7}{2}$$

So, $\frac{7}{2}$ is a zero of the polynomial $2x - 7$.

(iv) Solving the equation $h(y) = 0$, we get

$$2y = 0, \text{ which gives us } y = 0$$

So, 0 is a zero of the polynomial $2y$.

12. Find the zeroes of the polynomial $(x - 2)^2 - (x + 2)^2$.

Sol. Let $p(x) = (x - 2)^2 - (x + 2)^2$

As finding a zero of $p(x)$, is same as solving the equation $p(x) = 0$

$$\text{So, } p(x) = 0 \Rightarrow (x - 2)^2 - (x + 2)^2 = 0$$

$$\Rightarrow (x - 2 + x + 2)(x - 2 - x - 2) = 0$$

$$\Rightarrow 2x(-4) = 0 \Rightarrow -8x = 0 \Rightarrow x = 0$$

Hence, $x = 0$ is the only one zero of $p(x)$.

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial: $x^4 + 1$; $x + 1$.

Sol. By actual division, we have

$$\begin{array}{r}
 x^3 + x^2 + x + 1 \\
 x - 1 \overline{) x^4 + 1} \\
 \underline{-x^4 + x^3} \\
 x^3 + 1 \\
 \underline{-x^3 + x^2} \\
 x^2 + 1 \\
 \underline{-x^2 + x} \\
 x + 1 \\
 \underline{-x + 1} \\
 2
 \end{array}$$

14. By Remainder Theorem find the remainder, when $p(x)$ is divided by $g(x)$, where

(i) $p(x) = x^3 - 2x^2 - 4x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 - 3x^2 + 4x + 50$, $g(x) = x - 3$

(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$

(iv) $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$

Sol. (i) We have $g(x) = x + 1$

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

$$\text{Remainder} = p(-1)$$

$$= (-1)^3 - 2(-1)^2 - 4(-1) - 1 = -1 - 2 + 4 - 1 = 0$$

(ii) We have $g(x) = x - 3$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

$$\text{Remainder} = p(3)$$

$$= (3)^3 - 3(3)^2 + 4(3) + 50 = 27 - 27 + 12 + 50 = 62$$

(iii) We have $g(x) = 2x - 1$

$$\Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\begin{aligned}
 \text{Remainder} &= p\left(\frac{1}{2}\right) \\
 &= 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3 \\
 &= 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 7 - 3 \\
 &= \frac{1}{2} - 3 + 7 - 6 = \frac{1}{2} - 2 = \frac{-3}{2}
 \end{aligned}$$

$$(iv) g(x) = 0 \Rightarrow 1 - \frac{3}{2}x = 0; x = \frac{2}{3}$$

$$\begin{aligned}
 \text{Remainder} &= p\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{24}{9} + \frac{4}{3} - 4 \\
 &= \frac{8 - 72 + 36 - 108}{27} = \frac{-136}{27}
 \end{aligned}$$

15. Check whether $p(x)$ is a multiple of $g(x)$ or not:

$$(i) p(x) = x^3 - 5x^2 + 4x - 3, g(x) = x - 2$$

$$(ii) p(x) = 2x^3 - 11x^2 - 4x + 5, g(x) = 2x + 1$$

Sol. (i) $p(x)$ will be a multiple of $g(x)$ if $g(x)$ divides $p(x)$.

Now, $g(x) = x - 2$ gives $x = 2$

$$\begin{aligned}
 \text{Remainder} &= p(2) = (2)^3 - 5(2)^2 + 4(2) - 3 \\
 &= 8 - 5(4) + 8 - 3 = 8 - 20 + 8 - 3 \\
 &= -7
 \end{aligned}$$

Since remainder $\neq 0$, so $p(x)$ is not a multiple of $g(x)$.

(ii) $p(x)$ will be a multiple of $g(x)$ if $g(x)$ divides $p(x)$.

Now, $g(x) = 2x + 1$ give $x = -\frac{1}{2}$

$$\begin{aligned}
 \text{Remainder} &= p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 11\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 5 \\
 &= 2\left(-\frac{1}{8}\right) - 11\left(\frac{1}{4}\right) + 2 + 5 = \frac{-1}{4} - \frac{11}{4} + 7 \\
 &= \frac{-1 - 11 + 28}{4} = \frac{16}{4} = 4
 \end{aligned}$$

Since remainder $\neq 0$, so $p(x)$ is not a multiple of $g(x)$.

16. Show that:

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.

(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$.

Sol. (i) Let $p(x) = 69 + 11x - x^2 + x^3$, $g(x) = x + 3$.

$$g(x) = x + 3 = 0 \text{ gives } x = -3$$

$g(x)$ will be a factor of $p(x)$ if $p(-3) = 0$ (Factor theorem)

$$\begin{aligned} \text{Now, } p(-3) &= 69 + 11(-3) - (-3)^2 + (-3)^3 \\ &= 69 - 33 - 9 - 27 \\ &= 0 \end{aligned}$$

Since, $p(-3) = 0$, so, $g(x)$ is a factor of $p(x)$.

(ii) Let $p(x) = x + 2x^3 - 9x^2 + 12$ and $g(x) = 2x - 3$

$$g(x) = 2x - 3 = 0 \text{ gives } x = \frac{3}{2}$$

$g(x)$ will be a factor of $p(x)$ if $p\left(\frac{3}{2}\right) = 0$ (Factor theorem)

$$\begin{aligned} \text{Now, } p\left(\frac{3}{2}\right) &= \frac{3}{2} + 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 12 = \frac{3}{2} + 2\left(\frac{27}{8}\right) - 9\left(\frac{9}{4}\right) + 12 \\ &= \frac{3}{2} + \frac{27}{4} - \frac{81}{4} + 12 = \frac{6 + 27 - 81 + 48}{4} = \frac{0}{4} = 0 \end{aligned}$$

Since, $p\left(\frac{3}{2}\right) = 0$, so, $g(x)$ is a factor of $p(x)$.

17. Determine which of the following polynomials has $x - 2$ a factor:

(i) $3x^2 + 6x - 24$

(ii) $4x^2 + x - 2$

Sol. We know that if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

(i) Let $p(x) = 3x^2 + 6x - 24$

If $x - 2$ is a factor of $p(x) = 3x^2 + 6x - 24$, then $p(2)$ should be equal to 0.

$$\begin{aligned} \text{Now, } p(2) &= 3(2)^2 + 6(2) - 24 \\ &= 3(4) + 6(2) - 24 \\ &= 12 + 12 - 24 \\ &= 0 \end{aligned}$$

∴ By factor theorem, $(x - 2)$ is a factor of $3x^2 + 6x - 24$.

(ii) Let $p(x) = 4x^2 + x - 2$.

If $x - 2$ is a factor of $p(x) = 4x^2 + x - 2$, then $p(2)$ should be equal to 0.

$$\text{Now, } p(2) = 4(2)^2 + 2 - 2$$

$$\begin{aligned}
 &= 4(4) + 2 - 2 \\
 &= 16 + 2 - 2 \\
 &= 16 \neq 0
 \end{aligned}$$

$\therefore x - 2$ is not a factor of $4x^2 + x - 2$.

18. Show that $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

Sol. If $p - 1$ is a factor of $p^{10} - 1$, then $(1)^{10} - 1$ should be equal to zero.

Now, $(1)^{10} - 1 = 1 - 1 = 0$

Therefore, $p - 1$ is a factor of $p^{10} - 1$.

Again, if $p - 1$ is a factor of $p^{11} - 1$, then $(1)^{11} - 1$ should be equal to zero.

Now, $(1)^{11} - 1 = 1 - 1 = 0$

Therefore, $p - 1$ is a factor of $p^{11} - 1$.

Hence, $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

19. For what value of m is $x^3 - 2mx^2 + 16$ divisible by $x + 2$?

Sol. If $x^3 - 2mx^2 + 16$ is divisible by $x + 2$, then $x + 2$ is a factor of $x^3 - 2mx^2 + 16$.

Now, let $p(x) = x^3 - 2mx^2 + 16$.

As $x + 2 = x - (-2)$ is a factor of $x^3 - 2mx^2 + 16$

so $p(-2) = 0$

Now,
$$\begin{aligned}
 p(-2) &= (-2)^3 - 2m(-2)^2 + 16 \\
 &= -8 - 8m + 16 = 8 - 8m
 \end{aligned}$$

Now, $p(-2) = 0$

$\Rightarrow 8 - 8m = 0$

$\Rightarrow m = 8 \div 8$

$\Rightarrow m = 1$

Hence, for $m = 1$, $x + 2$ is a factor of $x^3 - 2mx^2 + 16$, so $x^3 - 2mx^2 + 16$ is completely divisible by $x + 2$.

20. If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a .

Sol. Let $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$

If $x - (-2a)$ is a factor of $p(x)$, then $p(-2a) = 0$

$$\begin{aligned}
 \therefore p(-2a) &= (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 \\
 &= -32a^5 + 32a^5 - 4a + 2a + 3 \\
 &= -2a + 3
 \end{aligned}$$

Now, $p(-2a) = 0$

$\Rightarrow -2a + 3 = 0$

$\Rightarrow a = \frac{3}{2}$

21. Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

Sol. Let $p(x) = 8x^4 + 4x^3 - 16x^2 + 10x + m$.

As $(2x - 1)$ is a factor of $p(x)$

$$\therefore p\left(\frac{1}{2}\right) = 0 \quad \text{[By factor theorem]}$$

$$\Rightarrow 8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m = 0$$

$$\Rightarrow 8\left(\frac{1}{16}\right) + 4\left(\frac{1}{8}\right) - 16\left(\frac{1}{4}\right) + 5 + m = 0$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0$$

$$\Rightarrow 2 + m = 0 \Rightarrow m = -2$$

22. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a .

Sol. Let $p(x) = ax^3 + x^2 - 2x + 4a - 9$.

As $(x + 1)$ is a factor of $p(x)$

$$\therefore p(-1) = 0 \quad \text{[By factor theorem]}$$

$$\Rightarrow a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0$$

$$\Rightarrow a(-1) + 1 + 2 + 4a - 9 = 0$$

$$\Rightarrow -a + 4a - 6 = 0$$

$$\Rightarrow 3a - 6 = 0 \Rightarrow 3a = 6 \Rightarrow a = 2$$

23. Factorise:

(i) $x^2 + 9x + 18$

(ii) $6x^2 + 7x - 3$

(iii) $2x^2 - 7x - 15$

(iv) $84 - 2r - 2r^2$

Sol. (i) In order to factorise $x^2 + 9x + 18$, we have to find two numbers p and q such that $p + q = 9$ and $pq = 18$.

Clearly, $6 + 3 = 9$ and $6 \times 3 = 18$.

So, we write the middle term $9x$ as $6x + 3x$.

$$\therefore x^2 + 9x + 18 = x^2 + 6x + 3x + 18$$

$$= x(x + 6) + 3(x + 6)$$

$$= (x + 6)(x + 3)$$

(ii) In order to factorise $6x^2 + 7x - 3$, we have to find two numbers p and q such that $p + q = 7$ and $pq = -18$.

Clearly, $9 + (-2) = 7$ and $9 \times (-2) = -18$.

So, we write the middle term $7x$ as $9x + (-2x)$, i.e., $9x - 2x$.

$$\begin{aligned}\therefore 6x^2 + 7x - 3 &= 6x^2 + 9x - 2x - 3 \\ &= 3x(2x + 3) - 1(2x + 3) \\ &= (2x + 3)(3x - 1)\end{aligned}$$

(iii) In order to factorise $2x^2 - 7x - 15$, we have to find two numbers p and q such that $p + q = -7$ and $pq = -30$.

Clearly, $(-10) + 3 = -7$ and $(-10) \times 3 = -30$.

So, we write the middle term $-7x$ as $(-10x) + 3x$.

$$\begin{aligned}\therefore 2x^2 - 7x - 15 &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x - 5) + 3(x - 5) \\ &= (x - 5)(2x + 3)\end{aligned}$$

(iv) In order to factorise $84 - 2r - 2r^2$, we have to find two numbers p and q such that $p + q = -2$ and $pq = -168$.

Clearly, $(-14) + 12 = -2$ and $(-14) \times 12 = -168$.

So, we write the middle term $-2r$ as $(-14r) + 12r$.

$$\begin{aligned}\therefore 84 - 2r - 2r^2 &= -2r^2 - 2r + 84 \\ &= -2r^2 - 14r + 12r + 84 \\ &= -2r(r + 7) + 12(r + 7) \\ &= (r + 7)(-2r + 12) \\ &= -2(r + 7)(r - 6) = -2(r - 6)(r + 7)\end{aligned}$$

24. Factorise:

(i) $2x^3 - 3x^2 - 17x + 30$ (ii) $x^3 - 6x^2 + 11x - 6$

(iii) $x^3 + x^2 - 4x - 4$ (iv) $3x^3 - x^2 - 3x + 1$

Sol. (i) Let $f(x) = 2x^3 - 3x^2 - 17x + 30$ be the given polynomial. The factors of the constant term $+30$ are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$. The factor of coefficient of x^3 is 2. Hence, possible rational roots of $f(x)$ are:

$$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$$

We have $f(2) = 2(2)^3 - 3(2)^2 - 17(2) + 30$
 $= 2(8) - 3(4) - 17(2) + 30$
 $= 16 - 12 - 34 + 30 = 0$

and $f(-3) = 2(-3)^3 - 3(-3)^2 - 17(-3) + 30$
 $= 2(-27) - 3(9) - 17(-3) + 30$
 $= -54 - 27 + 51 + 30 = 0$

So, $(x - 2)$ and $(x + 3)$ are factors of $f(x)$.

$\Rightarrow x^2 + x - 6$ is a factor of $f(x)$.

Let us now divide $f(x) = 2x^3 - 3x^2 - 17x + 30$ by $x^2 + x - 6$ to get the other factors of $f(x)$.

By long division, we have

$$\begin{array}{r} x^2 + x - 6 \overline{) 2x^3 - 3x^2 - 17x + 30} \quad 2x - 5 \\ \underline{2x^3 + 2x^2 - 12x} \\ -5x^2 - 5x + 30 \\ \underline{+ 5x^2 + 5x - 30} \\ 0 \end{array}$$

$$\therefore 2x^3 - 3x^2 - 17x + 30 = (x^2 + x - 6)(2x - 5)$$

$$\Rightarrow 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$$

$$\text{Hence, } 2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$$

(ii) Let $f(x) = x^3 - 6x^2 + 11x - 6$ be the given polynomial. The factors of the constant term -6 are $\pm 1, \pm 2, \pm 3$ and ± 6 .

$$\text{We have, } f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$$

$$\text{and, } f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$$

So, $(x - 1)$ and $(x - 2)$ are factors of $f(x)$.

$\Rightarrow (x - 1)(x - 2)$ is also a factor of $f(x)$.

$\Rightarrow x^2 - 3x + 2$ is a factor of $f(x)$.

Let us now divide $f(x) = x^3 - 6x^2 + 11x - 6$ by $x^2 - 3x + 2$ to get the other factors of $f(x)$.

By long division, we have

$$\begin{array}{r} x^2 - 3x + 2 \overline{) x^3 - 6x^2 + 11x - 6} \quad x - 3 \\ \underline{x^3 - 3x^2 + 2x} \\ -3x^2 + 9x - 6 \\ \underline{-3x^2 + 9x - 6} \\ 0 \end{array}$$

$$\therefore x^3 - 6x^2 + 11x - 6 = (x^2 - 3x + 2)(x - 3)$$

$$\Rightarrow x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

$$\text{Hence, } x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

(iii) Let $f(x) = x^3 + x^2 - 4x - 4$ be the given polynomial. The factors of the constant term -4 are $\pm 1, \pm 2, \pm 4$.

We have,

$$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$$

and,

$$f(2) = (2)^3 + (2)^2 - 4(2) - 4 = 8 + 4 - 8 - 4 = 0$$

So, $(x+1)$ and $(x-2)$ are factors of $f(x)$.

$\Rightarrow (x+1)(x-2)$ is also a factor of $f(x)$.

$\Rightarrow x^2 - x - 2$ is a factor of $f(x)$.

Let us now divide $f(x) = x^3 + x^2 - 4x - 4$ by $x^2 - x - 2$ to get the other factors of $f(x)$.

By long division, we have

$$\begin{array}{r} x^2 - x - 2 \overline{) x^3 + x^2 - 4x - 4} \quad x + 2 \\ \underline{-(x^3 - x^2 + 2x)} \\ 2x^2 - 2x - 4 \\ \underline{-(2x^2 - 2x - 4)} \\ 0 \end{array}$$

$$\therefore x^3 + x^2 - 4x - 4 = (x^2 - x - 2)(x + 2)$$

$$\Rightarrow x^3 + x^2 - 4x - 4 = (x+1)(x-2)(x+2)$$

$$\text{Hence, } x^3 + x^2 - 4x - 4 = (x-2)(x+1)(x+2)$$

(iv) Let $f(x) = 3x^3 - x^2 - 3x + 1$ be the given polynomial. The factors of the constant term +1 are ± 1 . The factor of coefficient of x^3 is 3. Hence,

possible rational roots of $f(x)$ are: $\pm \frac{1}{3}$.

We have,

$$f(1) = 3(1)^3 - (1)^2 - 3(1) + 1 = 3 - 1 - 3 + 1 = 0$$

and

$$f(-1) = 3(-1)^3 - (-1)^2 - 3(-1) + 1 = -3 - 1 + 3 + 1 = 0$$

So, $(x-1)$ and $(x+1)$ are factors of $f(x)$.

$\Rightarrow (x-1)(x+1)$ is also a factor of $f(x)$.

$\Rightarrow x^2 - 1$ is a factor of $f(x)$.

Let us now divide $f(x) = 3x^3 - x^2 - 3x + 1$ by $x^2 - 1$ to get the other factors of $f(x)$.

By long division, we have

$$\begin{array}{r} x^2 - 1 \overline{) 3x^3 - x^2 - 3x + 1} \quad 3x - 1 \\ \underline{-(3x^3 - 3x)} \\ -x^2 + 1 \\ \underline{-(-x^2 + 1)} \\ 0 \end{array}$$

$$\therefore 3x^3 - x^2 - 3x + 1 = (x^2 - 1)(3x - 1)$$

$$\Rightarrow 3x^3 - x^2 - 3x + 1 = (x-1)(x+1)(3x-1)$$

$$\text{Hence, } 3x^3 - x^2 - 3x + 1 = (x-1)(x+1)(3x-1)$$

25. Using suitable identity, evaluate the following:

(i) 103^3 (ii) 101×102 (iii) 999^2

Sol. (i) $103^3 = (100+3)^3$

Now, using identity $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$, we have

$$\begin{aligned} (100+3)^3 &= (100)^3 + (3)^3 + 3(100)(3)(100+3) \\ &= 1000000 + 27 + 900(100+3) \\ &= 1000000 + 27 + 90000 + 2700 \\ &= 1092727 \end{aligned}$$

(ii) $101 \times 102 = (100+1)(100+2)$

Now, using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$, we have

$$\begin{aligned} (100+1)(100+2) &= (100)^2 + (1+2)100 + (1)(2) \\ &= 10000 + (3)100 + 2 = 10000 + 300 + 2 \\ &= 10302 \end{aligned}$$

(iii) $(999)^2 = (1000-1)^2 = (1000)^2 - 2 \times (1000) \times 1 + 1^2$
 $= 1000000 - 2000 + 1$
 $= 998001$

26. Factorise the following:

(i) $4x^2 + 20x + 25$

(ii) $9y^2 - 66yz + 121z^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Sol. (i) We have,

$$\begin{aligned} 4x^2 + 20x + 25 &= (2x)^2 + 2(2x)(5) + (5)^2 \\ &= (2x+5)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2] \\ &= (2x+5)(2x+5) \end{aligned}$$

(ii) We have,

$$\begin{aligned} 9y^2 - 66yz + 121z^2 &= (-3y)^2 + 2(-3y)(11z) + (11z)^2 \\ &= (-3y+11z)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2] \\ &= (-3y+11z)(-3y+11z) \\ &= (3y-11z)(3y-11z) \end{aligned}$$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Using identity $a^2 - b^2 = (a+b)(a-b)$

$$\begin{aligned} &= \left[\left(2x + \frac{1}{3}\right) + \left(x - \frac{1}{2}\right)\right] \left[\left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right)\right] \\ &= \left(2x + \frac{1}{3} + x - \frac{1}{2}\right) \left(2x + \frac{1}{3} - x + \frac{1}{2}\right) = \left(3x - \frac{1}{6}\right) \left(x + \frac{5}{6}\right) \end{aligned}$$

27. Factorise the following:

(i) $9x^2 - 12x + 3$

(ii) $9x^2 - 12x + 4$

Sol. (i) $9x^2 - 12x + 3 = 9x^2 - 9x - 3x + 3$
 $= 9x(x - 1) - 3(x - 1)$
 $= (9x - 3)(x - 1)$
 $= 3(3x - 1)(x - 1)$

(ii) We have,

$$9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + (2)^2$$

$$= (3x - 2)^2 \quad [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= (3x - 2)(3x - 2)$$

28. Expand the following:

(i) $(4a - b + 2c)^2$

(ii) $(3a - 5b - c)^2$

(iii) $(-x + 2y - 3z)^2$

Sol. (i) We have,

$$(4a - b + 2c)^2 = (4a)^2 + (-b)^2 + (2c)^2 + 2(4a)(-b) + 2(-b)(2c) + 2(2c)(4a)$$

$$[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= 16a^2 + b^2 + 4c^2 - 8ab - 4bc + 16ca$$

(ii) We have,

$$(3a - 5b - c)^2 = (3a)^2 + (-5b)^2 + (-c)^2 + 2(3a)(-5b)$$

$$+ 2(-5b)(-c) + 2(-c)(3a)$$

$$[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= 9a^2 + 25b^2 + c^2 - 30ab + 10bc - 6ca.$$

(iii) $(-x + 2y - 3z)^2 = \{(-x) + 2y + (-3z)\}^2$
 $= (-x)^2 + (2y)^2 + (-3z)^2 + 2(-x)(2y) + 2(2y)(-3z)$
 $+ 2(-3z)(-x)$
 $= x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6zx$

29. Factorise the following:

(i) $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

(ii) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

(iii) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

Sol. (i) We have,

$$9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$$

$$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$$

$$= \{3x + 2y + (-4z)\}^2 \quad [\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$$

$$= (3x + 2y - 4z)^2 = (3x + 2y - 4z)(3x + 2y - 4z)$$

$$\begin{aligned}
 \text{(ii) } 25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz \\
 &= (-5x)^2 + (4y)^2 + (2z)^2 + 2(-5x)(4y) \\
 &\quad + 2(4y)(2z) + 2(2z)(-5x) \\
 &= (-5x + 4y + 2z)^2
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz \\
 &= (4x)^2 + (-2y)^2 + (3z)^2 + 2(4x)(-2y) + 2(-2y)(3z) + 2(3z)(4x) \\
 &= \{4x + (-2y) + 3z\}^2 \quad [\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\
 &= (a + b + c)^2] \\
 &= (4x - 2y + 3z)(4x - 2y + 3z)
 \end{aligned}$$

30. If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.

Sol. We know that

$$\begin{aligned}
 (a + b + c)^2 &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\
 \Rightarrow (a + b + c)^2 &= (a^2 + b^2 + c^2) + 2(ab + bc + ca) \\
 \Rightarrow 9^2 &= (a^2 + b^2 + c^2) + 2(26) \\
 &\quad [\text{Putting the values of } a + b + c \text{ and } ab + bc + ca] \\
 \Rightarrow 81 &= (a^2 + b^2 + c^2) + 52 \\
 \Rightarrow a^2 + b^2 + c^2 &= 81 - 52 = 29
 \end{aligned}$$

31. Expand the following:

$$\text{(i) } (3a - 2b)^3 \qquad \text{(ii) } \left(\frac{1}{x} + \frac{y}{3}\right)^3 \qquad \text{(iii) } \left(4 - \frac{1}{3x}\right)^3$$

Sol. (i) We have

$$\begin{aligned}
 (3a - 2b)^3 &= (3a)^3 - (2b)^3 - 3(3a)(2b)(3a - 2b) \\
 &\quad [\because (a - b)^3 = a^3 - b^3 - 3ab(a - b)] \\
 &= 27a^3 - 8b^3 - 18ab(3a - 2b) \\
 &= 27a^3 - 8b^3 - 54a^2b + 36ab^2
 \end{aligned}$$

$$\text{(ii) } \therefore (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\therefore \left(\frac{1}{x} + \frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3 \times \frac{1}{x} \times \frac{y}{3} \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x} \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x} = \frac{1}{x^3} + \frac{y}{x^2} + \frac{y^2}{3x} + \frac{y^3}{27}$$

(iii) We have,

$$\begin{aligned} \left(4 - \frac{1}{3x}\right)^3 &= (4)^3 - \left(\frac{1}{3x}\right)^3 - 3(4)\left(\frac{1}{3x}\right)\left(4 - \frac{1}{3x}\right) \\ &[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)] \\ &= 64 - \frac{1}{27x^3} - \frac{4}{x}\left(4 - \frac{1}{3x}\right) \\ &= 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2} \end{aligned}$$

32. Factorise the following:

(i) $1 - 64a^3 - 12a + 48a^2$ (ii) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

Sol. (i) We have,

$$\begin{aligned} 1 - 64a^3 - 12a + 48a^2 &= (1)^3 - (4a)^3 - 3(1)(4a)(1 - 4a) \\ &= (1 - 4a)^3 [\because a^3 - b^3 - 3ab(a-b) = (a-b)^3] \\ &= (1 - 4a)(1 - 4a)(1 - 4a) \end{aligned}$$

(ii) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

$$\begin{aligned} &= (2p)^3 + 3 \times (2p)^2 \times \frac{1}{5} + 3 \times (2p) \times \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 \\ &= (2p)^3 + \left(\frac{1}{5}\right)^3 + 3 \times (2p) \times \frac{1}{5} \left[2p + \frac{1}{5}\right] \end{aligned}$$

Now, using $a^3 + b^3 + 3ab(a+b) = (a+b)^3$

$$= \left(2p + \frac{1}{5}\right)^3 = \left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$$

33. Find the following products:

(i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$ (ii) $(x^2 - 1)(x^4 + x^2 + 1)$

Sol. (i) We have,

$$\begin{aligned} \left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right) &= \left(\frac{x}{2} + 2y\right)\left\{\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right\} \\ &= \left(\frac{x}{2}\right)^3 + (2y)^3 \quad [\because (a+b)(a^2 - ab + b^2) = a^3 + b^3] \\ &= \frac{x^3}{8} + 8y^3 \end{aligned}$$

(ii) We have,

$$\begin{aligned}(x^2-1)(x^4+x^2+1) &= (x^2-1)\{(x^2)^2+(x^2)(1)+(1)^2\} \\ &= (x^2)^3-(1)^3 \\ &[\because (a-b)(a^2+ab+b^2)=a^3-b^3] \\ &= x^6-1\end{aligned}$$

34. Factorise:

(i) $1+64x^3$

(ii) $a^3-2\sqrt{2}b^3$

Sol. (i) We have,

$$\begin{aligned}1+64x^3 &= (1)^3+(4x)^3 \\ &= (1+4x)\{(1)^2-(1)(4x)+(4x)^2\} \\ &[\because a^3+b^3=(a+b)(a^2-ab+b^2)] \\ &= (1+4x)(1-4x+16x^2) \\ &= (1+4x)(16x^2-4x+1) \\ &= (4x+1)(16x^2-4x+1)\end{aligned}$$

(ii) We have,

$$\begin{aligned}a^3-2\sqrt{2}b^3 &= (a)^3-(\sqrt{2}b)^3 \\ &= (a-\sqrt{2}b)\{(a)^2+(a)(\sqrt{2}b)+(\sqrt{2}b)^2\} \\ &[\because a^3-b^3=(a-b)(a^2+ab+b^2)] \\ &= (a-\sqrt{2}b)(a^2+\sqrt{2}ab+2b^2)\end{aligned}$$

35. Find the following product:

$$(2x-y+3z)(4x^2+y^2+9z^2+2xy+3yz-6xz)$$

Sol. We have,

$$\begin{aligned}(2x-y+3z)(4x^2+y^2+9z^2+2xy+3yz-6xz) \\ &= \{2x+(-y)+3z\}\{(2x)^2+(-y)^2+(3z)^2-(2x)(-y)-(-y)(3z)-(3z)(2x)\} \\ &= (2x)^3+(-y)^3+(3z)^3-3(2x)(-y)(3z) \\ &[\because (a+b+c)(a^2+b^2+c^2-ab-bc-ca)=a^3+b^3+c^3-3abc] \\ &= 8x^3-y^3+27z^3+18xyz\end{aligned}$$

36. Factorise:

(i) $a^3-8b^3-64c^3-24abc$

(ii) $2\sqrt{2}a^3+8b^3-27c^3+18\sqrt{2}abc$

Sol. (i) We have,

$$\begin{aligned}a^3-8b^3-64c^3-24abc \\ &= \{(a)^3+(-2b)^3+(-4c)^3-3(a)(-2b)(-4c)\} \\ &= \{a+(-2b)+(-4c)\}\{a^2+(-2b)^2+(-4c)^2-a(-2b) \\ &\quad -(-2b)(-4c)-(-4c)a\}\end{aligned}$$

$$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= (a - 2b - 4c)(a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ca)$$

(ii) We have,

$$2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$$

$$= \{(\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}a)(2b)(-3c)\}$$

$$= \{\sqrt{2}a + 2b + (-3c)\} \{(\sqrt{2}a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{2}a)(2b) - (2b)(-3c) - (-3c)(\sqrt{2}a)\}$$

$$= (\sqrt{2}a + 2b - 3c)(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}ab + 6bc + 3\sqrt{2}ca)$$

37. Without actually calculating the cubes, find the value of:

(i) $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ (ii) $(0.2)^3 - (0.3)^3 + (0.1)^3$

Sol. (i) Let $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = -\frac{5}{6}$

$$\therefore a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6}$$

$$= \frac{3 + 2 - 5}{6} = \frac{0}{6} = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\therefore \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3$$

$$= 3 \times \frac{1}{2} \times \frac{1}{3} \times \left(-\frac{5}{6}\right) = -\frac{5}{12}$$

(ii) We have,

$$(0.2)^3 - (0.3)^3 + (0.1)^3 = (0.2)^3 + (-0.3)^3 + (0.1)^3$$

Let $a = 0.2$, $b = -0.3$ and $c = 0.1$. Then,

$$a + b + c = 0.2 + (-0.3) + 0.1$$

$$= 0.2 - 0.3 + 0.1 = 0$$

$$\therefore a + b + c = 0$$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow (0.2)^3 + (-0.3)^3 + (0.1)^3 = 3(0.2)(-0.3)(0.1) = -0.018$$

$$\text{Hence, } (0.2)^3 + (-0.3)^3 + (0.1)^3 = -0.018$$

38. Without finding the cubes, factorise

$$(x-2y)^3 + (2y-3z)^3 + (3z-x)^3$$

Sol. Let $x-2y = a$, $2y-3z = b$ and $3z-x = c$

$$\therefore a + b + c = x - 2y + 2y - 3z + 3z - x = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$\begin{aligned} \text{Hence, } (x-2y)^3 + (2y-3z)^3 + (3z-x)^3 \\ = 3(x-2y)(2y-3z)(3z-x) \end{aligned}$$

39. Find the value of

(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$

(ii) $x^3 + 8y^3 - 36xy - 216$, when $x = 2y + 6$

Sol. (i) $x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3x \times y \times 4$
 $= (x + y + 4)(x^2 + y^2 + 4^2 - xy - 4y - 4x)$
 $[\because x + y = -4]$
 $= (0)(x^2 + y^2 + 4^2 - xy - 4y - 4x) = 0$

(ii) $x^3 - 8y^3 - 36xy - 216 = x^3 + (-2y)^3 + (-6)^3 - 3x(-2y)(-6)$
 $= (x - 2y - 6)$
 $[x^2 + (-2y)^2 + (-6)^2 - x(-2y) - (-2y)(-6) - (-6)x]$
 $= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$
 $= (0)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x) = 0$
 $[\because x = 2y + 6]$

40. Give possible experiments for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$.

Sol. Area : $4a^2 + 4a - 3$

Using the method of splitting the middle term, we first find two numbers whose sum is +4 and product is $4 \times (-3) = -12$.

Now, $+6 - 2 = +4$ and $(+6) \times (-2) = -12$

We split the middle term $4a$ as $4a = +6a - 2a$,

so that $4a^2 + 4a - 3 = 4a^2 + 6a - 2a - 3$
 $= 2a(2a + 3) - 1(2a + 3)$
 $= (2a - 1)(2a + 3)$

Now, area of rectangle = $4a^2 + 4a - 3$

Also, area of rectangle = length \times breadth and $4a^2 + 4a - 3 = (2a - 1)(2a + 3)$

So, the possible expressions for the length and breadth of the rectangle are length = $(2a - 1)$ and breadth = $(2a + 3)$ or, length = $(2a + 3)$ and breadth = $(2a - 1)$.

EXERCISE 2.4

1. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a .

Sol. Let $p(z) = ax^3 + 4z^2 + 3z - 4$

and $q(z) = z^3 - 4z + a$

As these two polynomials leave the same remainder, when divided by $z - 3$, then $p(3) = q(3)$.

$$\begin{aligned} \therefore p(3) &= a(3)^3 + 4(3)^2 + 3(3) - 4 \\ &= 27a + 36 + 9 - 4 \end{aligned}$$

$$\text{or } p(3) = 27a + 41$$

$$\begin{aligned} \text{and } q(3) &= (3)^3 - 4(3) + a \\ &= 27 - 12 + a = 15 + a \end{aligned}$$

$$\text{Now, } p(3) = q(3)$$

$$\Rightarrow 27a + 41 = 15 + a$$

$$\Rightarrow 26a = -26; a = -1$$

Hence, the required value of $a = -1$.

2. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves remainder 19. Also, find the remainder when $p(x)$ is divided by $x + 2$.

Sol. We know that if $p(x)$ is divided by $x + a$, then the remainder $= p(-a)$.

Now, $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ is divided by $x + 1$, then the remainder $= p(-1)$

$$\begin{aligned} \text{Now, } p(-1) &= (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7 \\ &= 1 - 2(-1) + 3(1) + a + 3a - 7 \\ &= 1 + 2 + 3 + 4a - 7 \\ &= -1 + 4a \end{aligned}$$

Also, remainder $= 19$

$$\therefore -1 + 4a = 19$$

$$\Rightarrow 4a = 20; a = 20 \div 4 = 5$$

Again, when $p(x)$ is divided by $x + 2$, then

$$\begin{aligned} \text{remainder} &= p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - a(-2) + 3a - 7 \\ &= 16 + 16 + 12 + 2a + 3a - 7 \\ &= 37 + 5a \\ &= 37 + 5(5) = 37 + 25 = 62 \end{aligned}$$

3. If both $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ are factors of $px^2 + 5x + r$, show that $p = r$.

Sol. Let $p(x) = px^2 + 5x + r$.

As $(x-2)$ is a factor of $p(x)$

$$\text{So, } p(2) = 0 \Rightarrow p(2)^2 + 5(2) + r = 0$$

$$\Rightarrow 4p + 10 + r = 0 \quad \dots(1)$$

Again, $\left(x - \frac{1}{2}\right)$ is a factor of $p(x)$,

$$\therefore p\left(\frac{1}{2}\right) = 0$$

$$\begin{aligned} \text{Now, } p\left(\frac{1}{2}\right) &= p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r \\ &= \frac{1}{4}p + \frac{5}{2} + r \end{aligned}$$

$$\therefore p\left(\frac{1}{2}\right) = 0 \Rightarrow \frac{1}{4}p + \frac{5}{2} + r = 0 \quad \dots(2)$$

From (1), we have $4p + r = -10$

From (2), we have $p + 10 + 4r = 0$

$$\Rightarrow p + 4r = -10$$

$$\therefore 4p + r = p + 4r \quad [\because \text{Each} = -10]$$

$$\therefore 3p = 3r \Rightarrow p = r$$

Hence, proved.

4. Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

Sol. We have,

$$\begin{aligned} x^2 - 3x + 2 &= x^2 - x - 2x + 2 \\ &= x(x-1) - 2(x-1) \\ &= (x-1)(x-2) \end{aligned}$$

$$\text{Let } p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$$

$$\text{Now, } p(1) = 2(1)^4 - 5(1)^3 + 2(1)^2 - 1 + 2 = 2 - 5 + 2 - 1 + 2 = 0$$

Therefore, $(x-1)$ divides $p(x)$

$$\begin{aligned} \text{and } p(2) &= 2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2 \\ &= 32 - 40 + 8 - 2 + 2 = 0 \end{aligned}$$

Therefore, $(x-2)$ divides $p(x)$.

$$\text{So, } (x-1)(x-2) = x^2 - 3x + 2 \text{ divides } 2x^4 - 5x^3 + 2x^2 - x + 2$$

5. Simplify $(2x-5y)^3 - (2x+5y)^3$.

Sol. We have,

$$(2x-5y)^3 - (2x+5y)^3$$

$$= \{(2x-5y) - (2x+5y)\} \{(2x-5y)^2 + (2x-5y)(2x+5y) + (2x+5y)^2\}$$

$$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$\begin{aligned}
 &= (2x - 5y - 2x - 5y)(4x^2 + 25y^2 - 20xy + 4x^2 - 25y^2 + 4x^2 + 25y^2 + 20xy) \\
 &= (-10y)(2x^2 + 25y^2) \\
 &= -120x^2y - 250y^3
 \end{aligned}$$

6. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

Sol. We have,

$$\begin{aligned}
 &(-z + x - 2y)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz) \\
 &= \{x + (-2y) + (-z)\} \{(x)^2 + (-2y)^2 + (-z)^2 - (x)(-2y) - (-2y)(-z) - (-z)(x)\} \\
 &= x^3 + (-2y)^3 + (-z)^3 - 3(x)(-2y)(-z) \\
 &\quad [\because (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc] \\
 &= x^3 - 8y^3 - z^3 - 6xyz
 \end{aligned}$$

7. If a, b, c are all non-zero and $a + b + c = 0$, prove that

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3.$$

Sol. We have a, b, c are all non-zero and $a + b + c = 0$, therefore

$$a^2 + b^2 + c^2 = 3abc$$

$$\text{Now, } \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc} = 3$$

8. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$

Sol. We know that,

$$\begin{aligned}
 a^3 + b^3 + c^3 - 3abc &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
 &= (a + b + c)[a^2 + b^2 + c^2 - (ab + bc + ca)] \\
 &= 5 \{a^2 + b^2 + c^2 - (ab + bc + ca)\} \\
 &= 5(a^2 + b^2 + c^2 - 10)
 \end{aligned}$$

Now, $a + b + c = 5$

Squaring both sides, we get

$$(a + b + c)^2 = 5^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 25$$

$$\therefore a^2 + b^2 + c^2 + 2(10) = 25$$

$$\Rightarrow a^2 + b^2 + c^2 = 25 - 20 = 5$$

$$\begin{aligned}
 \text{Now, } a^3 + b^3 + c^3 - 3abc &= 5(a^2 + b^2 + c^2 - 10) \\
 &= 5(5 - 10) = 5(-5) = -25
 \end{aligned}$$

Hence, proved.

9. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

Sol.

$$(a + b + c)^3 = [a + (b + c)]^3$$

$$= a^3 + 3a^2(b + c) + 3a(b + c)^2 + (b + c)^3$$

$$= a^3 + 3a^2b + 3a^2c + 3a(b^2 + 2bc + c^2)$$

$$+ (b^3 + 3b^2c + 3bc^2 + c^3)$$

$$= a^3 + 3a^2b + 3a^2c + 3ab^2 + 6abc + 3ac^2$$

$$+ b^3 + 3b^2c + 3bc^2 + c^3$$

$$= a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2c + 3b^2a$$

$$+ 3c^2a + 3c^2b + 6abc$$

$$= a^3 + b^3 + c^3 + 3a^2(b + c) + 3b^2(c + a)$$

$$+ 3c^2(a + b) + 6abc$$

Hence, above result can be put in the form

$$(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$$

$$\therefore (a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$$

