

8 ■ ■ ■

Quadrilaterals

EXERCISE 8.1

Write the correct answer in each of the following:

1. Three angles of a quadrilateral are 75° , 90° and 75° . The fourth angle is
 (a) 90° (b) 95° (c) 105° (d) 120°

Sol. Fourth angle of the quadrilateral
 $= 360^\circ - (75^\circ + 90^\circ + 75^\circ)$
 $= 360^\circ - 240^\circ$
 $= 120^\circ$

Hence, (d) is the correct answer.

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25° . The acute angle between the diagonals is
 (a) 55° (b) 50° (c) 40° (d) 25°

Sol. ABCD is a rectangle in which diagonal AC is inclined to one side AB of the rectangle at an angle of 25° .

Now,

$$AC = BD$$

[\because Diagonals of a rectangle are equal]



$$\therefore \frac{1}{2} AC = \frac{1}{2} BD$$

$$\Rightarrow OA = OB$$

In $\triangle AOB$, we have $OA = OB$

$$\therefore \angle OBA = \angle OAB = 25^\circ$$

$$\therefore \angle AOB = 180^\circ - (25^\circ + 25^\circ) = 130^\circ$$

$\angle AOB$ and $\angle AOD$ form angles of a linear pair.

$$\therefore \angle AOB + \angle AOD = 180^\circ$$

$$\Rightarrow \angle AOD = 180^\circ - 130^\circ = 50^\circ$$

Hence, the acute angle between the diagonals is 50° .

Therefore, (b) is the correct answer.

3. ABCD is a rhombus such that $\angle ACB = 40^\circ$. Then $\angle ADB$ is
 (a) 40° (b) 45° (c) 50° (d) 60°

Sol. ABCD is a rhombus such that

$$\angle ACB = 40^\circ.$$

We know that diagonals of a rhombus bisect each other at right angles.

In right $\triangle BOC$, we have

$$\begin{aligned}\angle OBC &= 180^\circ - (\angle BOC + \angle BCO) \\ &= 180^\circ - (90^\circ + 40^\circ) = 50^\circ\end{aligned}$$

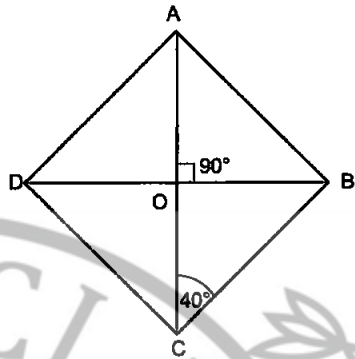
$$\therefore \angle DBC = \angle OBC = 50^\circ$$

Now,

$$\angle ADB = \angle DBC \quad [\text{Alt. int. } \angle s]$$

$$\therefore \angle ADB = 50^\circ \quad [\because \angle DBC = 50^\circ]$$

Hence, (c) is the correct answer.



4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in a order, is a rectangle, if

- (a) PQRS is a rectangle
- (b) PQRS is a parallelogram
- (c) diagonals of PQRS are perpendicular
- (d) diagonals of PQRS are equal

Sol. If diagonals of PQRS are perpendicular.

Hence, (c) is the correct answer.

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if

- (a) PQRS is a rhombus
- (b) PQRS is a parallelogram
- (c) diagonals of PQRS are perpendicular
- (d) diagonals of PQRS are equal

Sol. If diagonals of PQRS are equal.

Hence, (d) is the correct answer.

6. If angles A, B, C and D of the quadrilateral ABCD, taken in a order, are in the ratio 3 : 7 : 6 : 4, then ABCD is a

- (a) rhombus
- (b) parallelogram
- (c) trapezium
- (d) kite

Sol. As angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4, so let the angles A, B, C and D be $3x$, $7x$, $6x$ and $4x$.

Now, sum of the angles of a quadrilateral is 360° .

$$3x + 7x + 6x + 4x = 360^\circ$$

$$\Rightarrow 20x = 360^\circ \quad \Rightarrow \quad x = 360^\circ \div 20 = 18^\circ$$

So, the angles A, B, C and D of quadrilateral ABCD are $3 \times 18^\circ$, $7 \times 18^\circ$, $6 \times 18^\circ$ and $4 \times 18^\circ$

i.e., 54° , 126° , 108° and 72°

Now, AD and BC are two lines which are cut by a transversal CD such that the sum of angles $\angle C$ and $\angle D$ on the same side of transversal is $\angle C + \angle D = 108^\circ + 72^\circ = 180^\circ$

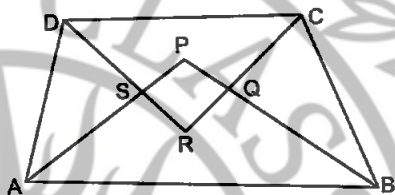
$\therefore AD \parallel BC$

So, ABCD is a quadrilateral in which one pair of opposite sides are parallel. Hence, ABCD is a trapezium.

Hence, (c) is the correct answer.

7. If bisectors of $\angle A$ and $\angle B$ of a quadrilateral ABCD intersect each other at P, of $\angle B$ and $\angle C$ at Q, of $\angle C$ and $\angle D$ at R and of $\angle D$ and $\angle A$ at S, then PQRS is a

- (a) rectangle (b) rhombus
(c) parallelogram
(d) quadrilateral whose opposite angles are supplementary



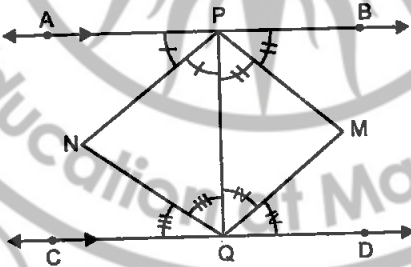
Sol. PQRS is a quadrilateral whose opposite angles are supplementary.

Hence, (d) is the correct answer.

8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form

- (a) a square (b) a rhombus
(c) a rectangle (d) any other parallelogram

Sol.



PNQM is a rectangle. Hence, (c) is the correct answer.

9. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is

- (a) a rhombus (b) a rectangle
(c) a square (d) any parallelogram

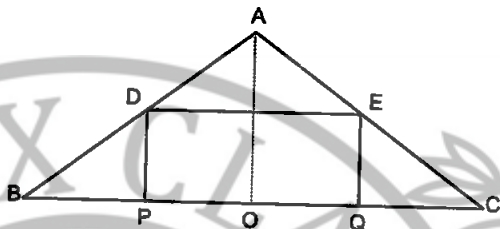
Sol. The figure will be a rectangle.

Hence, (b) is the correct answer.

10. D and E are the mid-points of the sides AB and AC of $\triangle ABC$ and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is

- (a) a square
(b) a rectangle
(c) a rhombus
(d) a parallelogram

Sol. Since the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it, so



$$\therefore DE = \frac{1}{2} BC \text{ and } DE \parallel BC$$

Similarly,

$$DP = \frac{1}{2} AO \text{ and } DP \parallel AO$$

and

$$EQ = \frac{1}{2} AO \text{ and } EQ \parallel AO$$

$$\therefore DP = EQ \quad \left[\because \text{Each} = \frac{1}{2} AO \right]$$

and $DP \parallel EQ$ [$\because DP \parallel AO$ and $EQ \parallel AO$]

Now, DEQP is a quadrilateral in which one pair of its opposite sides is equal and parallel.

Therefore, quadrilateral DEQP is a parallelogram.

Hence, (d) is the correct answer.

11. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only, if

- (a) ABCD is a rhombus
(b) diagonals of ABCD are equal
(c) diagonals of ABCD are equal and perpendicular
(d) diagonals of ABCD are perpendicular.

Sol. If diagonals of ABCD are equal and perpendicular.

Hence, (c) is the correct answer.

12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O, if $\angle DAC = 32^\circ$ and $\angle AOB = 70^\circ$ then $\angle DBC$ is equal to

- (a) 24°
(b) 86°
(c) 38°
(d) 32°

Sol. AD is parallel to BC and AC cuts them,

$$\angle DAC = \angle ACB \text{ [Alt. int. } \angle\text{s]}$$

$$\angle DAC = 32^\circ \quad \text{[Given]}$$

$$\therefore \angle ACB = 32^\circ$$

Now, in $\triangle BOC$, CO is produced to A

$$\therefore \text{Ext. } \angle BOA = \angle OCB + \angle OBC \text{ [By exterior angle theorem]}$$

$$\Rightarrow 70^\circ = 32^\circ + \angle OBC$$

$$\therefore \angle OBC = 70^\circ - 32^\circ = 38^\circ$$

Hence, $\angle DBC = 38^\circ$.

Therefore, (c) is the correct answer.

13. Which of the following is not true for a parallelogram?

- (a) Opposite sides are equal.
- (b) Opposite angles are equal.
- (c) Opposite angles are bisected by the diagonals.
- (d) Diagonals bisect each other.

Sol. Opposite angles are bisected by the diagonals. This is not true for a parallelogram.

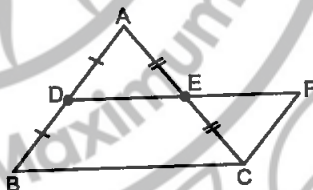
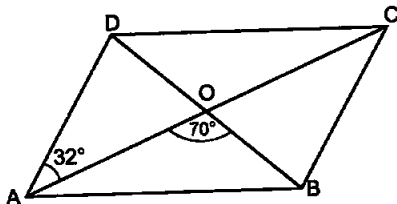
Hence, (c) is the correct answer.

14. D and E are the mid-points of the sides AB and AC respectively of $\triangle ABC$. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is

- (a) $\angle DAE = \angle EFC$
- (b) $AE = EF$
- (c) $DE = EF$
- (d) $\angle ADE = \angle ECF$

Sol. We need $DE = EF$.

Hence, (c) is the correct answer.



EXERCISE 8.2

1. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If $OA = 3 \text{ cm}$ and $OD = 2 \text{ cm}$, determine the lengths of AC and BD.

Sol. We know that diagonals of a parallelogram bisect each other.

$$\therefore AC = 2 \times OA = 2 \times 3 \text{ cm} = 6 \text{ cm}$$

$$\text{and } BD = 2OD = 2 \times 2 \text{ cm} = 4 \text{ cm}$$

Hence, lengths of AC and BD are 6 cm and 4 cm respectively.

2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Sol. This statement not true. Diagonals of a parallelogram bisect each other.

3. Can the angles 110° , 80° , 70° and 95° be the angles of a quadrilateral? Why or why not?

Sol. Sum of these angles $110^\circ + 80^\circ + 70^\circ + 95^\circ = 355^\circ$

But, sum of the angles of a quadrilateral is always 360° . Hence, 110° , 80° , 70° and 95° can not be the angles of a quadrilateral.

4. In quadrilateral ABCD, $\angle A + \angle D = 180^\circ$. What special name can be given to this quadrilateral?

Sol. In quadrilateral ABCD, $\angle A + \angle D = 180^\circ$ i.e., the sum of two consecutive angles is 180° . So, pair of opposite sides AB and CD are parallel.

Therefore, quadrilateral ABCD is a trapezium. Hence, special name which can be given to this quadrilateral ABCD is trapezium.

5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Sol. All the angles of a quadrilateral are equal. Also, the sum of angles of a quadrilateral is 360° . Therefore, each angle of a quadrilateral is 90° . So, the given quadrilateral is a rectangle.

Hence, special name given to this quadrilateral is rectangle.

6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.

Sol. The given statement is not true. Diagonals of a rectangle need not to be perpendicular.

7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

Sol. No, because then the sum of four angles of the quadrilateral will be more than 360° whereas sum of four angles of a quadrilateral is always equal to 360° .

8. In $\triangle ABC$, $AB = 5$ cm, $BC = 8$ cm and $CA = 7$ cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE.

Sol. In $\triangle ABC$, $AB = 5$ cm, $BC = 8$ cm and $CA = 7$ cm. D and E are respectively the mid-points of AB and BC.

$$\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 7 \text{ cm} = 3.5 \text{ cm} \quad [\text{Using the mid-point theorem}]$$

9. In the given figure, it is given that BDEF and FDCE are parallelograms. Can you say that $BD = CD$? Why or why not?

Sol. BDEF is a parallelogram.

$$\therefore BD = EF \quad \dots(1)$$

[Opposite sides of a parallelogram are equal]

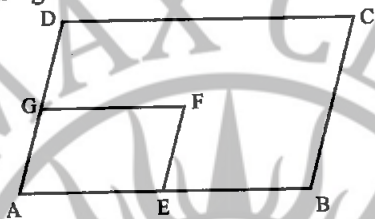
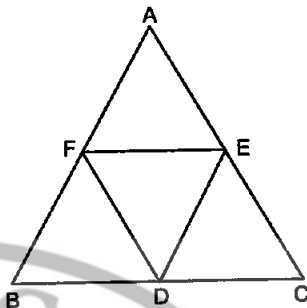
FDCE is a parallelogram

$$\therefore CD = EF \quad \dots(2)$$

From (1) and (2), we get

$$BD = CD$$

10. In the given figure, ABCD and AEFG are two parallelograms.



If $\angle C = 55^\circ$, determine $\angle F$.

Sol. We know that opposite angles of a parallelogram are equal.

In parallelogram ABCD, we have

$$\angle A = \angle C$$

But, $\angle C = 55^\circ$

$$\therefore \angle A = 55^\circ$$

Now, in parallelogram AEFG, we have

$$\angle F = \angle A = 55^\circ$$

Hence, $\angle F = 55^\circ$.

11. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.

Sol. We know that an acute angle is less than 90° . All the angles of a quadrilateral cannot be acute angles because then angle sum of a quadrilateral will be less than a 360° , whereas angle sum of a quadrilateral is 360° .

12. Can all the angles of a quadrilateral be right angles? Give reason for your answer.

Sol. Yes, all the angles of a quadrilateral can be right angles. Angle sum of a quadrilateral will be 360° , which is true.

13. Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35^\circ$, determine $\angle B$.

Sol. As the diagonals of a quadrilateral ABCD bisect each other, so ABCD is a parallelogram.

Now, ABCD is a parallelogram

$$\therefore \angle A + \angle B = 180^\circ$$

[\because Adjacent angles of a parallelogram are supplementary]

$$\begin{aligned} \therefore 35^\circ + \angle B &= 180^\circ \\ \Rightarrow \angle B &= 180^\circ - 35^\circ = 145^\circ \end{aligned}$$

14. Opposite angles of a quadrilateral ABCD are equal. If $AB = 4$ cm, determine CD.

Sol. Since opposite angles of a quadrilateral are equal, so it is parallelogram. Now, ABCD is a parallelogram, so $AB = CD$.

[\because Opposite sides of a parallelogram are equal]

But, $AB = 4$ cm, therefore $CD = 4$ cm.

Hence, $CD = 4$ cm.

EXERCISE 8.3

1. One angle of a quadrilateral is of 108° and the remaining three angles are equal. Find each of the three equal angles.

Sol. One angle of a quadrilateral is of 108° and let each of the three remaining equal angles be x° .

As the sum of the angles of a quadrilateral is 360° .

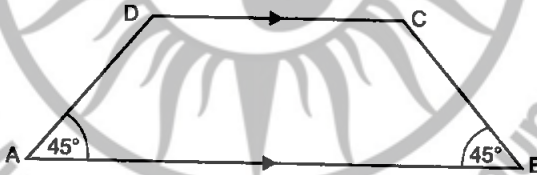
$$\therefore 108^\circ + x + x + x = 360^\circ \Rightarrow 3x = 360^\circ - 108^\circ = 252^\circ$$

$$\therefore x = 252^\circ \div 3 = 84^\circ$$

Hence, each of three angles be 84° .

2. ABCD is a trapezium in which $AB \parallel DC$ and $\angle A = \angle B = 45^\circ$. Find angles C and D of the trapezium.

Sol. ABCD is a trapezium in which $AB \parallel DC$.



Now, $AB \parallel DC$ and AD is a transversal.

$$\therefore \angle A + \angle D = 180^\circ$$

[\because Sum of interior angles on the same side of the transversal is 180°]

$$\Rightarrow 45^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 45^\circ = 135^\circ$$

Similarly, $\angle B + \angle C = 180^\circ$

$$\Rightarrow 45^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 45^\circ = 135^\circ$$

Hence, $\angle A = \angle B = 45^\circ$ and $\angle C = \angle D = 135^\circ$.

3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60° . Find the angles of the parallelogram.

Sol. In quadrilateral PBQD,

$$\begin{aligned} \angle 1 + \angle 2 + \angle B + \angle 3 &= 360^\circ \\ \Rightarrow 60^\circ + 90^\circ + \angle B + 90^\circ &= 360^\circ \\ \Rightarrow \angle B + 240^\circ &= 360^\circ \\ \Rightarrow \angle B &= 360^\circ - 240^\circ \\ \Rightarrow \angle B &= 120^\circ \end{aligned}$$

Now,

$\angle ADC = \angle B = 120^\circ$
 [\because Opposite angles of a parallelogram are equal]

$$\angle A + \angle B = 180^\circ$$

[\because Sum of consecutive interior angles is 180°]

$$\begin{aligned} \Rightarrow \angle A + 120^\circ &= 180^\circ \\ \Rightarrow \angle A &= 180^\circ - 120^\circ \\ \Rightarrow \angle A &= 60^\circ \end{aligned}$$

But,

$$\angle C = \angle A = 60^\circ$$

[\because Opposite angles of a parallelogram are equal]

4. ABCD is a rhombus in which altitude from D to side AB bisects AB.
 Find the angles of the rhombus.

Sol. In $\triangle APD$ and $\triangle BPD$, we have

$$AP = BP \quad \text{[Given]}$$

$$\angle 1 = \angle 2 \quad \text{[\because Each equal to } 90^\circ\text{]}$$

$$PD = PD \quad \text{[Common side]}$$

So, by SAS criterion of congruence,
 we have

$$\triangle APD \cong \triangle BPD$$

$$\therefore \angle A = \angle 3$$

$$\text{But, } \angle 3 = \angle 4$$

[\because Diagonals bisect opposite angles of a rhombus]

$$\Rightarrow \angle A = \angle 3 = \angle 4 \quad \dots (1)$$

Now, $AD \parallel BC$

So, $\angle A + \angle ABC = 180^\circ$ [\because Sum of consecutive interior angles is 180°]

$$\Rightarrow \angle A + \angle 3 + \angle 4 = 180^\circ$$

$$\Rightarrow \angle A + \angle A + \angle A = 180^\circ \quad \text{[Using (1)]}$$

$$\Rightarrow 3\angle A = 180^\circ$$

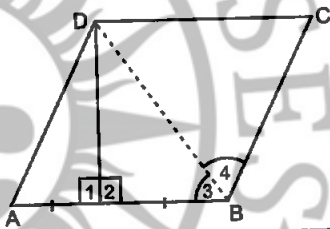
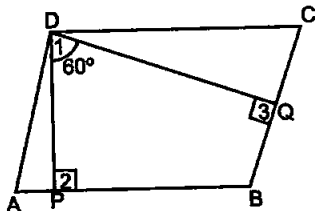
$$\Rightarrow \angle A = \frac{180^\circ}{3} = 60^\circ$$

Now,

$$\begin{aligned} \angle ABC &= \angle 3 + \angle 4 \\ &= 60^\circ + 60^\circ \end{aligned}$$

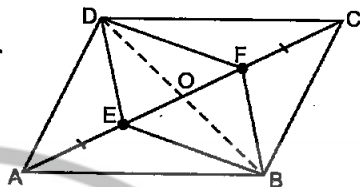
$$= 120^\circ \quad \text{[\because Opposite angles of a rhombus are equal]}$$

$$\therefore \angle ADC = \angle ABC = 120^\circ \quad \text{[Same reason as above]}$$



5. E and F are points on diagonal AC of a parallelogram ABCD such that $AE = CF$. Show that BFDE is a parallelogram.

Sol. Given : A parallelogram ABCD:
E and F are points on diagonal AC of parallelogram ABCD such that $AE = CF$.



To prove : BFDE is a parallelogram.

Proof: ABCD is a parallelogram.

$$\begin{aligned} \therefore OD &= OB && \dots (1) && [\because \text{Diagonals of parallelogram bisect each other}] \\ OA &= OC && \dots (2) && [\text{Same reason as above}] \\ AE &= CF && \dots (3) && [\text{Given}] \end{aligned}$$

Subtracting (3) from (2), we get

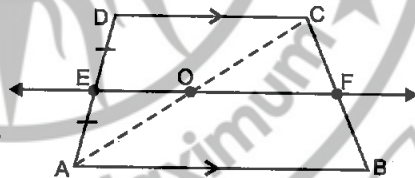
$$\begin{aligned} OA - AE &= OC - CF \\ \Rightarrow OE &= OF && \dots (4) \end{aligned}$$

\therefore BFDE is a parallelogram. [$\because OD = OB$ and $OE = OF$]

Hence, proved.

6. E is the mid-point of the side AD of the trapezium ABCD with $AB \parallel DC$. A line through E drawn parallel to AB intersect BC at F. Show that F is the mid-point of BC. [Hint: Join AC]

Sol. Given : A trapezium ABCD in which $AB \parallel DC$ and E is the mid-point of the side AD. Also, $EF \parallel AB$.



To prove : F is the mid-point of BC.

Construction : Join AC which intersects EF at O.

Proof: In $\triangle ADC$, E is the mid-point of AD and $EF \parallel DC$.

$$[\because EF \parallel AB \text{ and } DC \parallel AB \Rightarrow AB \parallel EF \parallel DC]$$

\therefore O is the mid-point of AC. [Converse of mid-point theorem]

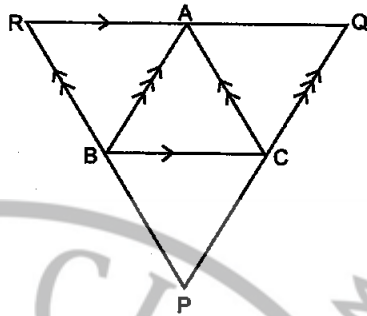
Now, in $\triangle CAB$, O is the mid-point of AC and $OF \parallel AB$.

\Rightarrow OF bisects BC.

Or, F is the mid-point of BC.

Hence, proved.

7. Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a $\triangle ABC$ as shown in the



given figure. Show that $BC = \frac{1}{2} QR$.

Sol. Given : Triangles ABC and PQR in which $AB \parallel QP$, $BC \parallel RQ$ and $CA \parallel PR$.

To prove : $BC = \frac{1}{2} QR$

Proof : Quadrilateral RBCA is a parallelogram.

[$\because RA \parallel BC$ and $BR \parallel CA$]

$\therefore RA = BC$... (1) [\because Opposite sides of a parallelogram]

Now, quadrilateral BCQA is a parallelogram.

$\therefore AQ = BC$... (2) [\because Opposite sides of a parallelogram]

Adding (1) and (2), we get

$$RA + AQ = 2BC$$

$$\Rightarrow QR = 2BC$$

$$\Rightarrow BC = \frac{1}{2} QR$$

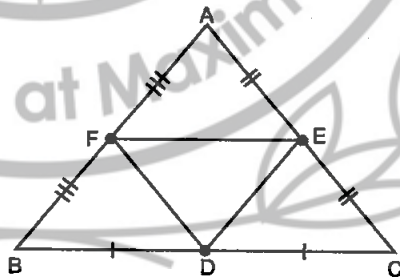
Hence, proved.

8. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that $\triangle DEF$ is also an equilateral triangle.

Sol. Given : $\triangle ABC$ is an equilateral triangle. D, E and F are the mid-points of the sides BC, CA and AB, respectively of $\triangle ABC$.

To prove : $\triangle DEF$ is an equilateral triangle.

Proof : FE joins mid-points of sides AB and AC respectively.



$$\therefore FE = \frac{1}{2} BC$$

... (1) [Mid-point theorem]

$$\text{Similarly, } DE = \frac{1}{2} BC$$

... (2) [Mid-point theorem]

$$DF = \frac{1}{2} AC \quad \dots (3) \text{ [Mid-point theorem]}$$

But, $AB = BC = CA \quad \dots (4) \text{ [Sides of an equilateral } \Delta ABC]$

From (1), (2), (3) and (4), we have

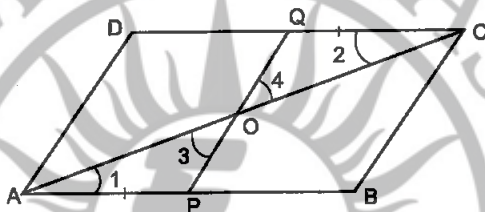
$$DE = EF = FD$$

$\therefore \Delta DEF$ is an equilateral triangle.

Hence, proved.

9. Points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that $AP = CQ$. Show that AC and PQ bisect each other.

Sol. Points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that $AP = CQ$.



In ΔAOP and ΔCOQ , we have

$$AP = CQ$$

[Given]

$$\angle 1 = \angle 2$$

[Alt. int \angle s are equal]

$$\angle 3 = \angle 4$$

[Vertically opp. \angle s]

$$\therefore \Delta AOP \cong \Delta COQ$$

[By AAS Congruence rule]

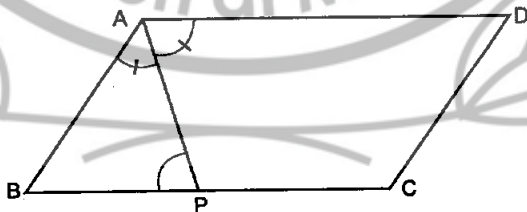
$$\therefore OA = OC \text{ and } OP = OQ$$

[CPCT]

Hence, AC and PQ bisect each other.

10. In the given figure, P is the mid-point of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. Prove that $AD = 2CD$.

Sol. $\angle BAP = \angle DAP = \frac{1}{2} \angle A \quad \dots (1)$



Since ABCD is a parallelogram, we have

$$\angle A + \angle B = 180^\circ \quad \dots (2)$$

[\because Sum of interior angles on the same side of transversal is 180°]

In $\triangle ABP$, we have

$$\angle BAP + \angle B + \angle APB = 180^\circ$$

$$\Rightarrow \frac{1}{2}\angle A + 180^\circ - \angle A + \angle APB = 180^\circ \quad [\text{Using (1) and (2)}]$$

$$\Rightarrow \angle APB = \frac{1}{2}\angle A \quad \dots(3)$$

From (1) and (3), we get

$$\begin{aligned} \angle BAP &= \angle APB \\ BP &= AB \end{aligned} \quad \dots(4)$$

[\because Sides opposite to equal angles are equal]

Since opposite sides of a parallelogram are equal, we have

$$AD = BC \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$$

$$\Rightarrow \frac{1}{2}AD = BP \quad [\because P \text{ is the mid-point of } BC]$$

$$\Rightarrow \frac{1}{2}AD = AB \quad [\because \text{From (4), } BP = AB]$$

Since opposite sides of a parallelogram are equal, we have

$$\frac{1}{2}AD = CD \Rightarrow AD = 2CD$$

Hence, proved.

EXERCISE 8.4

1. A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Sol. $\triangle ABC$ is an isosceles right triangle with $AB = BC$. A square $BFED$ is inscribed in triangle ABC so that $\angle B = \text{common} = 90^\circ$.

In $\triangle ADE$ and $\triangle EFC$, we have

$$DE = EF \quad \dots(1)$$

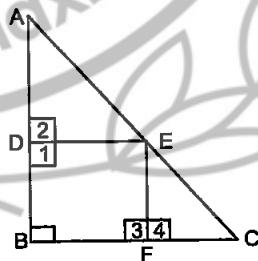
[\because Sides of a square are equal]

$$\angle 1 + \angle 2 = 180^\circ$$

[Linear pair axiom]

$$\Rightarrow 90^\circ + \angle 2 = 180^\circ \quad [\because \text{Each angle of a square} = 90^\circ]$$

$$\Rightarrow \angle 2 = 90^\circ$$



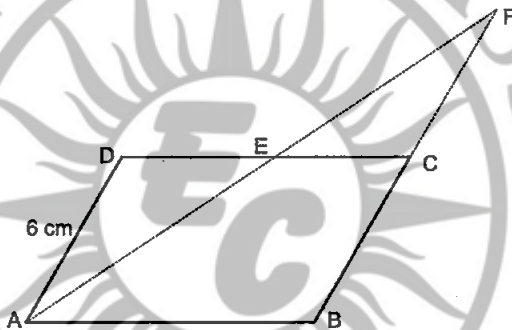
Similarly, $\angle 4 = 90^\circ$
 $\therefore \angle 2 = \angle 4$... (2) [\because Each $= 90^\circ$]
 Now, $AB = BC$ [Given]
 $\therefore \angle C = \angle A$... (3)
 [\because Angles opp. to equal sides are equal]

From (1), (2) and (3), we get
 $\triangle ADE \cong \triangle EFC$ [By AAS Congruence rule]

Hence, $AE = EC$ [CPCT]

2. In a parallelogram ABCD, $AB = 10$ cm and $AD = 6$ cm. The bisector of $\angle A$ meets DC in E. AE and BC produced meet at F. Find the length of CF.

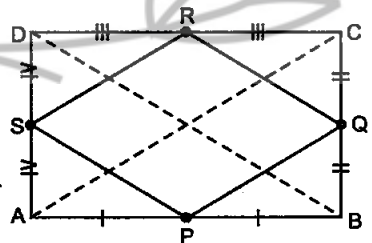
Sol. ABCD is a parallelogram, in which $AB = 10$ cm and $AD = 6$ cm. The bisector of $\angle A$ meets DC in E. AE and BC produced meet at F.



$\angle BAE = \angle EAD$... (1) [\because AE bisects $\angle A$]
 $\angle EAD = \angle EFB$... (2) [Alt. \angle s]
 $\Rightarrow \angle BAE = \angle EFB$ [From (1) and (2)]
 $\therefore BF = AB$ [\because Sides opposite to equal \angle s are equal]
 $\Rightarrow BF = 10$ cm [\because $AB = 10$ cm]
 $\Rightarrow BC + CF = 10$ cm $\Rightarrow 6$ cm + $CF = 10$ cm
 [\because $BC = AD = 6$ cm, opposite sides of a ||gm]
 $\Rightarrow CF = 10$ cm - 6 cm = 4 cm

3. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which $AC = BD$. Prove that PQRS is a rhombus.

Sol. Given: A quadrilateral ABCD in which $AC = BD$ and P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a rhombus.

Proof: In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.

That is, PQ joins mid-points of sides AB and BC.

$$\therefore PQ \parallel AC \quad \dots(1)$$

$$\text{and} \quad PQ = \frac{1}{2} AC \quad \dots(2) \text{ [Mid-point theorem]}$$

In $\triangle ADC$, R and S are the mid-points of CD and AD respectively.

$$\therefore SR \parallel AC \quad \dots(3)$$

$$\text{and} \quad SR = \frac{1}{2} AC \quad \dots(4) \text{ [Mid-point theorem]}$$

From (1) and (3), we get $PQ \parallel SR$ | From (2) and (4), we get

$$\Rightarrow PQ = SR$$

\Rightarrow PQRS is a parallelogram.

In $\triangle DAB$, SP joins mid-points of sides DA and AB respectively.

$$\therefore SP = \frac{1}{2} BD \quad \dots(5) \text{ [Mid-point theorem]}$$

$$AC = BD \quad \dots(6) \text{ [Given]}$$

From equations (2), (5) and (6), we get

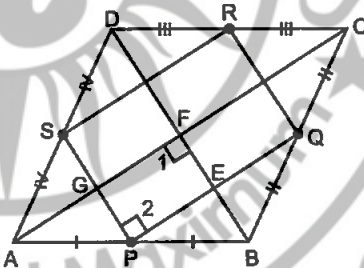
$$SP = PQ$$

\therefore Parallelogram PQRS is a rhombus.

Hence, proved.

4. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that $AC \perp BD$. Prove that PQRS is a rectangle.

Sol. Given: A quadrilateral ABCD in which $AC \perp BD$ and P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a rectangle.

Proof: In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.

That is, PQ joins mid-points of sides AB and BC.

$$\therefore PQ \parallel AC \quad \dots(1)$$

$$\text{and} \quad PQ = \frac{1}{2} AC \quad \dots(2) \text{ [Mid-point theorem]}$$

In $\triangle ADC$, R and S are the mid-points of CD and AD respectively.

$$\therefore SR \parallel AC \quad \dots(3)$$

and $SR = \frac{1}{2}AC$... (4) [Mid-point theorem]

From (1) and (3), we get

$$PQ \parallel SR$$

\Rightarrow PQRS is a parallelogram.

$$PQ \parallel AC$$

From (2) and (4), we get

$$PQ = SR$$

[Proved above]

\Rightarrow PE \parallel GF

In $\triangle ABD$, PS joins mid-points of sides AB and AD respectively.

\therefore PS \parallel BD [Mid-point theorem]

\Rightarrow PG \parallel EF

\Rightarrow PEFG is a parallelogram [\because PE \parallel GF and PG \parallel EF]

$\Rightarrow \angle 1 = \angle 2$ [\because Opposite angles of a parallelogram are equal]

But, $\angle 1 = 90^\circ$ [\because AC \perp BD]

$\therefore \angle 2 = 90^\circ$

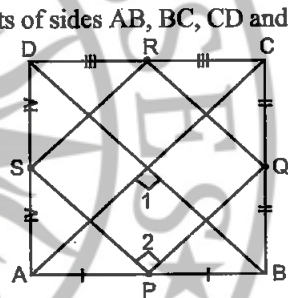
\Rightarrow Parallelogram PQRS is a rectangle.

5. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC \perp BD. Prove that PQRS is a square.

Sol. Given : A quadrilateral ABCD in which AC = BD and AC \perp BD. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD.

To prove : PQRS is a square.

Proof : Parallelogram PQRS is a rectangle.



[Same as in Q4]

$$PQ = \frac{1}{2}AC$$

... (1) [Proved as in Q4]

PS joins mid-points of sides AB and AD respectively.

$$PS = \frac{1}{2}BD$$

... (2) [Mid-point theorem]

$$AC = BD$$

... (3) [Given]

From (1), (2) and (3), we get

$$PS = PQ$$

\Rightarrow Rectangle PQRS is a square.

6. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Sol. ABCD is a parallelogram and diagonal AC bisects $\angle A$. We have to show that ABCD is a rhombus.

$$\angle 1 = \angle 2$$

... (1) [\because AC bisects $\angle A$]

$$\angle 2 = \angle 4$$

...(2) [\because Alt. interior angles]

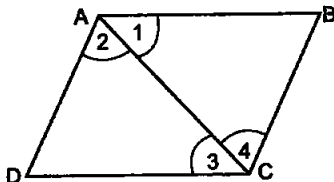
From (1) and (2), we get

$$\angle 1 = \angle 4$$

Now, in $\triangle ABC$, we have

$$\angle 1 = \angle 4 \text{ [Proved above]}$$

$$\therefore BC = AB$$



[\because Sides opp. to equal \angle s are equal]

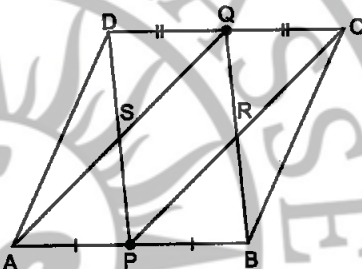
Also, $AB = DC$ and $AD = BC$ [\because Opposite sides of a parallelogram are equal]

So, ABCD is a parallelogram in which its sides $AB = BC = CD = AD$.

Hence, ABCD is a rhombus.

7. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PQRS is a parallelogram.

Sol. Given : A parallelogram ABCD in which P and Q are the mid-points of the sides AB and CD respectively. AQ intersects DP at S and BQ intersects CP at R.



To prove : PQRS is a parallelogram.

Proof: $DC \parallel AB$ [\because Opposite sides of a parallelogram are parallel]

$\Rightarrow AP \parallel QC$

$DC = AB$ [\because Opposite sides of a parallelogram are equal]

$$\Rightarrow \frac{1}{2}DC = \frac{1}{2}AB$$

$\Rightarrow QC = AP$ [\because P is mid-point of AB and Q is mid-point of CD]

\Rightarrow APCQ is a parallelogram. [$\because AP \parallel QC$ and $QC = AP$]

$\therefore AQ \parallel PC$ [\because Opposite sides of a \parallel gm are parallel]

$\Rightarrow SQ \parallel PR$

Similarly, $SP \parallel QR$

\therefore Quadrilateral PQRS is a parallelogram.

Hence, proved.

8. ABCD is a quadrilateral in which $AB \parallel DC$ and $AD = BC$. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.

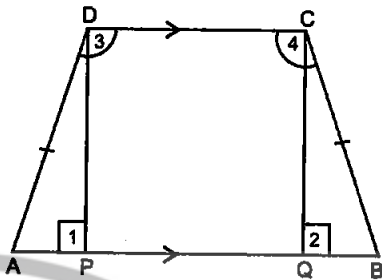
Sol. Given : A quadrilateral ABCD in which $AB \parallel DC$ and $AD = BC$.

To prove : $\angle A = \angle B$ and $\angle C = \angle D$

Construction : Draw $DP \perp AB$ and $CQ \perp AB$.

Proof : In $\triangle APD$ and $\triangle BQC$, we have

$$\begin{aligned} \angle 1 &= \angle 2 \\ [\because \text{Each equal to } 90^\circ] \\ AD &= BC \quad [\text{Given}] \\ DP &= CQ \end{aligned}$$



[Distance between parallel lines]

So, by RHS criterion of congruence, we have

$$\triangle APD \cong \triangle BQC$$

$$\therefore \angle A = \angle B \quad [\text{CPCT}]$$

Now, $DC \parallel AB$

$$\angle A + \angle 3 = 180^\circ \quad \dots(1) \quad [\because \text{Sum of consecutive interior angles is } 180^\circ]$$

$$\angle B + \angle 4 = 180^\circ \quad \dots(2) \quad [\text{Same reason}]$$

From (1) and (2), we get

$$\angle A + \angle 3 = \angle B + \angle 4$$

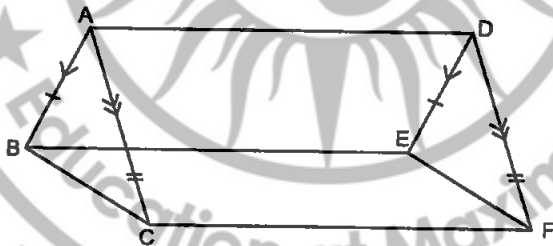
$$\Rightarrow \angle 3 = \angle 4 \quad [\because \angle A = \angle B]$$

$$\Rightarrow \angle C = \angle D$$

Hence, proved.

9. In the given figure, $AB \parallel DE$, $AB = DE$, $AC \parallel DF$ and $AC = DF$. Prove that $BC \parallel EF$ and $BC = EF$.

Sol.



Given : $AB \parallel DE$, $AB = DE$, $AC \parallel DF$ and $AC = DF$

To prove : $BC \parallel EF$ and $BC = EF$

Proof : $AC \parallel DF$ [Given]

and $AC = DF$ [Given]

\therefore ACFD is a parallelogram.

$\Rightarrow AD \parallel CF$... (1) [\because Opposite sides of a \parallel gm are parallel]

and $AD = CF$... (2) [\because Opposite sides of a \parallel gm are equal]

Now, $AB \parallel DE$ [Given]

and $AB = DE$ [Given]

∴ ABED is a parallelogram.

⇒ AD ∥ BE ... (3) [∵ Opposite sides of a ∥ gm are parallel]

and AD = BE ... (4) [∵ Opposite sides of a ∥ gm are equal]

From (1) and (3), we get

$$CF \parallel BE$$

And, from (2) and (4), we get

$$CF = BE$$

∴ BCFE is a parallelogram.

⇒ BC ∥ EF [∵ Opposite sides of a ∥ gm are parallel]

and BC = EF [∵ Opposite sides of a ∥ gm are equal]

Hence, proved.

10. E is the mid-point of a median AD of $\triangle ABC$ and BE is produced to meet AC at F. Show that $AF = \frac{1}{3} AC$.

meet AC at F. Show that $AF = \frac{1}{3} AC$.

Sol. Given : A $\triangle ABC$ in which E is the mid-point of median AD and BE is produced to meet AC at F.

To prove : $AF = \frac{1}{3} AC$

Construction: Draw $DG \parallel BF$ intersecting AC at G.

Proof: In $\triangle ADG$, E is the mid-point of AD and $EF \parallel DG$.

∴ $AF = FG$... (1) [Converse of mid-point theorem]

In $\triangle FBC$, D is the mid-point of BC and $DG \parallel BF$.

∴ $FG = GC$... (2) [Converse of mid-point theorem]

From equations (1) and (2), we get

$$AF = FG = GC \quad \dots (3)$$

But, $AC = AF + FG + GC$

⇒ $AC = AF + AF + AF$ [Using (3)]

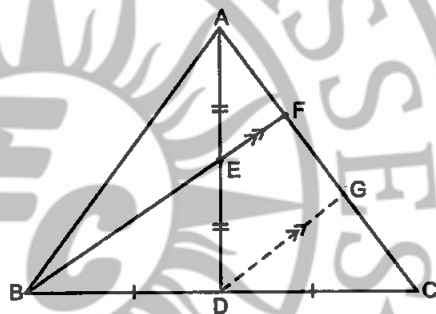
⇒ $AC = 3 AF$

⇒ $AF = \frac{1}{3} AC$

Hence, proved.

11. Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square.

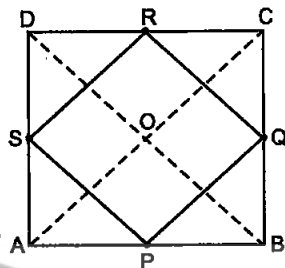
Sol. Given: A square ABCD in which P, Q, R, S are the mid-points of sides AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a square.

Construction: Join AC and BD.

Proof: In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.



$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(1)$$

In $\triangle ADC$, R and S are the mid-points of CD and AD respectively.

$$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2} AC \quad \dots(2)$$

From eqs. (1) and (2), we get

$$PQ \parallel RS \text{ and } PQ = RS \quad \dots(3)$$

Thus, in quadrilateral PQRS one pair of opposite sides are equal and parallel.

Hence, PQRS is a parallelogram.

Since ABCD is a square.

$$\therefore AB = BC = CD = DA$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD \text{ and } \frac{1}{2} AB = \frac{1}{2} BC$$

$$\Rightarrow PB = RC \text{ and } BQ = CQ$$

Thus, in $\triangle PBQ$ and $\triangle RCQ$, we have

$$PB = RC$$

$$BQ = CQ$$

$$[\Rightarrow PB = CR \text{ and } BQ = CQ]$$

$$\text{and, } \angle PBQ = \angle RCQ$$

$$[\because \text{Each equal to } 90^\circ]$$

So, by SAS criterion of congruence, we have

$$\triangle PBQ \cong \triangle RCQ$$

$$\Rightarrow PQ = QR \quad \text{[CPCT]} \quad \dots(4)$$

From (3) and (4), we have

$$PQ = QR = RS$$

But, PQRS is a parallelogram

$$\therefore QR = PS$$

$$\text{So, } PQ = QR = RS = PS \quad \dots(5)$$

$$\text{Now, } PQ \parallel AC \quad \text{[From (1)]}$$

$$\Rightarrow PM \parallel NO \quad \dots(6)$$

Since P and S are the mid-points of AB and AD respectively.

$$PS \parallel BD$$

$$\Rightarrow PN \parallel MO \quad \dots(7)$$

Thus, in quadrilateral PMON, we have

$$PM \parallel NO \quad \text{[From (6)]}$$

$$\text{and } PN \parallel MO \quad \text{[From (7)]}$$

So, PMON is a parallelogram.

$$\Rightarrow \angle MPN = \angle MON$$

$$\Rightarrow \angle MPN = \angle BOA$$

$$\Rightarrow \angle MPN = 90^\circ$$

$$[\because \angle MON = \angle BOA]$$

$[\because \text{Diagonals of square are } \perp$

$$\therefore AC \perp BD \Rightarrow \angle BOA = 90^\circ]$$

$$\Rightarrow \angle QPS = 90^\circ$$

Thus, PQRS is a quadrilateral such that $PQ = QR = RS = SP$ and $\angle QPS = 90^\circ$,

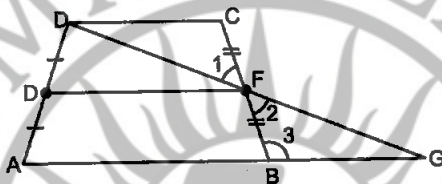
Hence, PQRS is a square.

12. E and F are respectively the mid-points of the non-parallel sides AD and

BC of a trapezium ABCD. Prove that $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$.

[Hint: Join BE and produce it to meet CD produced at G]

Sol.



Given : A trapezium ABCD in which E and F are respectively the mid-points of the non-parallel sides AD and BC.

To prove : $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$

Construction: Join DF and produce it to intersect AB produced at G.

Proof : In $\triangle CDF$ and $\triangle BFG$, we have

$$\therefore DC \parallel AB$$

$$\angle C = \angle 3$$

[Alternate interior angles]

$$CF = BF$$

[Given]

$$\angle 1 = \angle 2$$

[Vertically opposite angles]

So, by ASA criterion of congruence, we have

$$\triangle CDF \cong \triangle BFG$$

$$\therefore CD = BG$$

[CPCT]

EF joins mid-points of sides AD and GD respectively

$$\therefore EF \parallel AG$$

[\because Mid-point theorem]

$$\Rightarrow EF \parallel AB$$

$$\text{So, } EF = \frac{1}{2}AG$$

[Mid-point theorem]

$$\Rightarrow EF = \frac{1}{2}(AB + BG)$$

$$\Rightarrow EF = \frac{1}{2}(AB + CD)$$

[$\because CD = BG$]

Hence, proved.

13. Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

Sol. Given : A parallelogram ABCD in which bisectors of angles A,B,C,D intersect at P,Q,R,S to form a quadrilateral PQRS.

To prove : PQRS is a rectangle.

Proof : Since ABCD is a parallelogram.

Therefore, $AB \parallel DC$

Now, $AB \parallel DC$ and transversal AD intersects them at D and A respectively.

Therefore,

$$\angle A + \angle D = 180^\circ [\because \text{Sum of consecutive interior angles is } 180^\circ]$$

$$\Rightarrow \frac{1}{2} \angle A + \frac{1}{2} \angle D = 90^\circ$$

$$\Rightarrow \angle DAS + \angle ADS = 90^\circ \quad \dots(1)$$

[\because DS and AS are bisectors of $\angle A$ and $\angle D$ respectively]

But, in $\triangle DAS$, we have

$$\angle DAS + \angle ASD + \angle ADS = 180^\circ$$

[\because Sum of the angles of a triangle is 180°]

$$\Rightarrow \angle 90^\circ + \angle ASD = 180^\circ \quad [\text{Using (1)}]$$

$$\Rightarrow \angle ASD = 90^\circ$$

$$\Rightarrow \angle PSR = 90^\circ \quad [\because \angle ASD \text{ and } \angle PSR \text{ are vertically opposite angles } \therefore \angle PSR = \angle ASD]$$

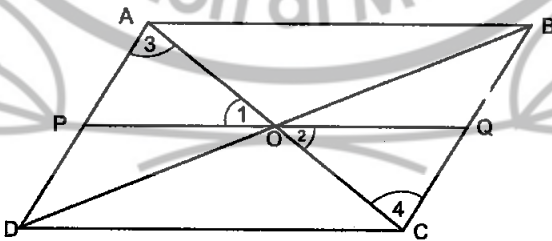
Similarly, we can prove that

$$\angle SRQ = 90^\circ, \angle RQP = 90^\circ \text{ and } \angle SPQ = 90^\circ$$

Hence, PQRS is a rectangle.

14. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD . Show that PQ is bisected at O.

Sol. ABCD is a parallelogram. Its diagonals AC and BD bisect each other at O. PQ passes through the point of intersection O of its diagonal AC and BD.



In $\triangle AOP$ and $\triangle COQ$, we have

$$\angle 3 = \angle 4$$

[Alternate int. $\angle s$]

$$OA = OC$$

$$\angle 1 = \angle 2$$

$$\therefore \Delta AOP \cong \Delta COQ$$

$$\text{So, } OP = OQ$$

Hence, PQ is bisected at O.

[Diagonals of a ||gm bisect each other]

[Vertically opposite angles]

[By ASA Congruence rule]

[CPCT]

15. ABCD is a rectangle in which diagonal BD bisects $\angle B$. Show that ABCD is a square.

Sol. Given : A rectangle ABCD in which diagonal BD bisects $\angle B$.

To prove : ABCD is a square.

Proof : $DC \parallel AB$

[\because Opposite sides of a rectangle are parallel]

$$\Rightarrow \angle 4 = \angle 1 \quad \dots(1) \text{ [Alternate interior angles]}$$

$$\text{Similarly, } \angle 3 = \angle 2 \quad \dots(2) \text{ [Alternate interior angles]}$$

$$\text{and } \angle 1 = \angle 2 \quad \dots(3) \quad \text{[Given]}$$

From equations (1), (2) and (3), we get

$$\angle 3 = \angle 4$$

In ΔBDA and ΔBDC , we have

$$\angle 1 = \angle 2 \quad \text{[Given]}$$

$$BD = BD \quad \text{[Common side]}$$

$$\angle 3 = \angle 4 \quad \text{[Proved above]}$$

So, by ASA criterion of congruence, we have

$$\Delta BDA \cong \Delta BDC$$

$$\therefore AB = BC \quad \text{[CPCT]}$$

So, ABCD is a square.

Hence, proved.

16. D, E and F are respectively the mid-points of the sides AB, BC and CA of a triangle ABC. Prove that by joining these mid-points D, E and F, the triangle ABC is divided into four congruent triangles.

Sol. Given : A ΔABC and ΔDEF which is formed by joining the mid-points D, E and F of the sides AB, BC and CA of ΔABC .

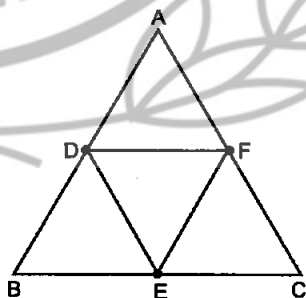
To prove : $\Delta DEF \cong \Delta EDB \cong \Delta CFE \cong \Delta FAD$

Proof : DF joins mid-points of sides AB and AC respectively of ΔABC .

$$\therefore DF \parallel BC \text{ [Mid-point theorem]}$$

$$\Rightarrow DF \parallel BE$$

Similarly, $EF \parallel BD$



So, quadrilateral BEFD is a parallelogram.

$\Rightarrow \triangle DEF \cong \triangle EDB$... (1) [\because A diagonal of a parallelogram divides it into two congruent triangles]

Similarly, $\triangle DEF \cong \triangle CFE$... (2)

and $\triangle DEF \cong \triangle FAD$... (3)

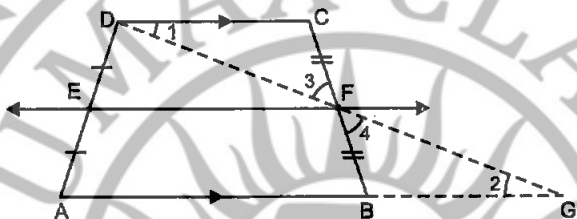
From equations (1), (2) and (3), we get

$$\triangle DEF \cong \triangle EDB \cong \triangle CFE \cong \triangle FAD$$

Hence, proved.

17. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

Sol.



Given : A trapezium ABCD in which E and F are the mid-points of sides AD and BC respectively.

To prove : $EF \parallel AB \parallel DC$

Construction : Join DF and produce it to intersect AB produced at G.

Proof : In $\triangle DCF$ and $\triangle GBF$, we have

$$\angle 1 = \angle 2 \quad [\text{Alternate interior angles because } DC \parallel BG]$$

$$\angle 3 = \angle 4 \quad [\text{Vertically opposite angles}]$$

$$CF = BF \quad [\because F \text{ is the mid-point of } BC]$$

So, by AAS criterion of congruence, we have

$$\triangle DCF \cong \triangle GBF$$

$$\therefore DF = GF \quad [\text{CPCT}]$$

In $\triangle DAG$, EF joins mid-points of sides DA and DG respectively.

$$\therefore EF \parallel AG \quad [\text{Mid-point theorem}]$$

$$\Rightarrow EF \parallel AB$$

$$\text{But, } AB \parallel DC \quad [\text{Given}]$$

$$\therefore EF \parallel AB \parallel DC$$

Hence, proved.

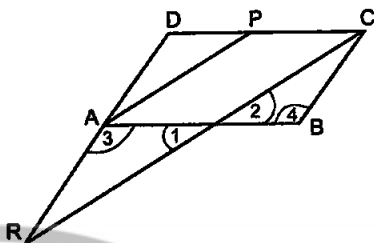
18. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that $DA = AR$ and $CQ = QR$.

Sol. ABCD is a parallelogram. P is the mid-point of CD. CR which intersects AB at Q is parallel to AP.

In $\triangle DCR$, P is the mid-point of CD and $AP \parallel CR$,

∴ A is the mid-point of DR, i.e.,
AD = AR.

[∵ The line drawn through the mid-point of one side of a triangle parallel to another side intersects the third side at its mid-point.]



In $\triangle ARQ$ and $\triangle BCQ$, we have

$$AR = BC \quad [∵ AD = AR \text{ (proved above) and } AD = BC]$$

$$\angle 1 = \angle 2 \quad [\text{Vertically opposite angles}]$$

$$\angle 3 = \angle 4 \quad [\text{Alt. } \angle\text{s}]$$

$$\therefore \triangle ARQ \cong \triangle BCQ \quad [\text{By AAS Congruence rule}]$$

$$CQ = QR \quad [\text{CPCT}]$$

Hence, DA = AR and CQ = QR is proved.

