13 Surface Areas and Volumes

EXERCISE 13.1

Write the correct answer in each of the following:

1. The radius of a sphere is $2r$, then its volume will be

(a) $\frac{4}{3}\pi r^3$ (b) $4\pi r^3$ (c) $\frac{8\pi r^3}{3}$ (d) $\frac{32}{3}\pi r^3$

Sol. $\therefore$ Volume of sphere $= \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi \times 8r^3 = \frac{32}{3}\pi r^3$

Hence, (*d*) is the correct answer.

2. The total surface area of cube is 96 cm^2. The volume of the cube is:

(a) 8 cm^3 (b) 512 cm^3 (c) 64 cm^3 (d) 27 cm^3

Sol. Total surface area of cube = $6(\text{edge})^2$

$\therefore 6(\text{edge})^2 = 96 \quad \Rightarrow (\text{edge})^2 = 96 \div 6 = 16$

$\therefore$ Edge $= +\sqrt{16} = 4$ cm

Hence, volume of cube = $(\text{edge})^3 = (4)^3 = 64$ cm^3

Hence, (*c*) is the correct answer.

3. A cone is 8.4 cm high and radius of its base is 2.1 cm. It is melted and recast into sphere. The radius of the sphere is:

(a) 4.2 cm (b) 2.1 cm (c) 2.4 cm (d) 1.6 cm

Sol. Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2.1)^2 \times 8.4$

Now, volume of sphere $= \frac{4}{3}\pi r_1^3$

$$\therefore \quad \frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi(2.1)^2 \times 8.4$$

$$\Rightarrow \quad 4r_1^3 = (2.1)^2 \times 8.4$$

$$\therefore \quad r_1^3 = \frac{(2.1)^2 \times 8.4}{4} = (2.1)^3$$

$$\Rightarrow \quad r_1 = \sqrt[3]{(2.1)^3} = 2.1 \text{ cm}$$

$\therefore$ Radius of sphere = 2.1 cm.

Hence, (*b*) is the correct answer.

4. In a cylinder, radius is doubled and height is halved, curved surface will be

(*a*) halved (*b*) doubled (*c*) same (*d*) four times

Sol. Curved surface area of cylinder $= 2\pi rh$

When radius is doubled and height is halved, then curved surface area

$$= 2\pi(2r) \times \frac{h}{2} = 2\pi rh$$

So, the curved surface area will be same.

Hence, (*c*) is the correct answer.

5. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is

(*a*) $2\pi r(l+r)$ (*b*) $\pi r\left(l+\frac{r}{4}\right)$ (*c*) $\pi r(l+r)$ (*d*) $2\pi rl$

Sol. Total surface area of cone = Area of the base + Curved surface area of cone

$$= \pi\left(\frac{r}{2}\right)^2 + \pi\left(\frac{r}{2}\right) \times 2l = \frac{\pi r}{2}\left(\frac{r}{2}+2l\right)$$

$$= \frac{\pi r}{4}(r+4l) = \pi r\left(l+\frac{r}{4}\right)$$

Hence, (*b*) is the correct answer.

6. The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is :

(*a*) 10 : 17 (*b*) 20 : 27 (*c*) 17 : 27 (*d*) 20 : 37.

Sol. Let the radii of two cylinders be $2r$ and $3r$ respectively and their heights are in the ratio $5h$ and $3h$. Then,

$$\frac{V_1}{V_2} = \frac{\pi(2r)^2(5h)}{\pi(3r)^2(3h)} = \frac{4r^2 \times 5h}{9r^2 \times 3h} = \frac{20}{27}$$

Hence, (*b*) is the correct answer.

7. The lateral surface area of a cube is 256 m². The volume of the cube is

(a) 512 m³ (b) 64 m³ (c) 216 m³ (d) 256 m³

Sol. Lateral surface area of cube $= 4(\text{edge})^2$

$\therefore$ $4(\text{edge})^2 = 256$

$\Rightarrow$ $(\text{edge})^2 = 256 \div 4 = 64$

$\Rightarrow$ $\text{edge} = +\sqrt{64} = 8\text{ m}$

$\therefore$ $\text{Volume of cube} = (\text{edge})^3 = (8)^3 = 512\text{ m}^3$

Hence, (*a*) is the correct answer.

8. The number of planks of dimension (4 m × 50 cm × 20 cm) that can be stored in a pit, which is 16 m long, 12 m wide and 4 m deep is

(*a*) 1900 (*b*) 1920 (*c*) 1800 (*d*) 1840

Sol. Volume of pit = $(16 \times 12 \times 4)\ m^3$

Volume of a plank = $(4 \times 0.5 \times 0.2)\ m^3$

$$\text{Required number of planks} = \frac{\text{Volume of pit}}{\text{Volume of a plank}}$$

$$= \frac{16 \times 12 \times 4}{4 \times 0.5 \times 0.2} = 1920$$

Hence, (*b*) is the correct answer.

9. The length of the longest pole that can be put in a room of dimensions (10 m × 10 m × 5 m) is

(*a*) 15 m (*b*) 16 m (*c*) 10 m (*d*) 12 m

Sol. Length of the longest pole = $\sqrt{l^2 + b^2 + h^2}$

$$= \sqrt{10^2 + 10^2 + 5^2}$$

$$= \sqrt{100 + 100 + 25} = \sqrt{225} = 15\ m$$

Hence, (*a*) is the correct option.

10. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is

(*a*) 1 : 4 (*b*) 1 : 3 (*c*) 2 : 3 (*d*) 2 : 1

Sol. Balloon is hemispherical in shape.

Surface area of hemispherical balloon of radius is = $3\pi r^2$

$$\therefore \quad \frac{S_1}{S_2} = \frac{3\pi r_1^2}{3\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{(6)^2}{(12)^2} = \frac{36}{144} = 1 : 4$$

Therefore, the ratio of the surface areas of two balloons = 1 : 4.

Hence, (*a*) is the correct answer.

EXERCISE 13.2

Write True or False and justify your answer in each of the following:

1. The volume of a sphere is equal to the two-third of the volume of a cylinder whose height and diameter are equal to the diameter of sphere.

Sol. Let the radius of the sphere be *r*.

As it is given that height and diameter of cylinder are equal to the diameter of sphere.

So, the radius of the cylinder is *r* and the height of the cylinder = 2*r*

Now, volume of sphere = $\frac{2}{3}$ volume of cylinder

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{2}{3}(\pi r^2 \times 2r) = \frac{4}{3}\pi r^3$$

Hence, the given statement is true.

2. If the radius of a right circular cone is halved and height is doubled, the volume will remain unchanged.

Sol. Let the original radius of the cone be r and height be h.

$$\text{The volume of cone} = \frac{1}{3}\pi r^2 h$$

Now, when radius of a right circular cone is halved and height is doubled, then

$$V = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times 2h = \frac{1}{3}\pi \times \frac{r^2}{4} \times 2h = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

We see that the volume becomes half of the original volume.
Hence, the given statement is false.

3. In a right circular cone, height, radius and slant height do not always be sides of a right triangle.

Sol. In a right circular cone, height (h), radius (r) and slant height (l) are always the sides of a right triangle i.e $l^2 = r^2 + h^2$
Hence, the given statement is false.

4. If the radius of cylinder is doubled and its curved surface area is not changed, the height must be halved.

Sol. Let r be the radius and h the height of the cylinder.
Then the curved surface area = $2\pi rh$
When the radius is doubled and the curved surface area is not changed, the height must be halved.

Then the curved surface area = $2\pi(2r)\frac{h}{2}$ which is equal to $2\pi rh$.

Hence, the given statement is true.

5. The volume of the largest right circular cone that can be fitted in a cube whose edge is $2r$ equals to volume of the hemisphere of radius r.

Sol. The height of the largest cone is $2r$ that can be fitted in a cube whose edge is $2r$.

$$\text{Its volume} = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$$

But $\frac{2}{3}\pi r^3$ is the volume of a hemisphere of radius r.

Hence, the given statement is true.

6. A cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is three times the volume of the cone.

Sol. Let r be the radius and h be the height of the cylinder and a right circular cone.

Now, volume of cylinder $= \pi r^2 h$

and volume of cone $= \frac{1}{3}\pi r^2 h$

Clearly, Volume of cylinder = 3(Volume of cone)

Hence, the given statement is true.

7. A cone, a hemisphere and a cylinder stand on equal bases and same height. The ratio of their volumes is 1 : 2 : 3.

Sol. A cone, a hemisphere and a cylinder stand on equal bases and same height.

Let radius of each of a cone, a hemisphere and cylinder be r (equal bases)

Height of hemisphere $= r$,

so the height of the cone $= r$

and height of cylinder $= r$.

Now, volume of cone, $V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times r = \frac{1}{3}\pi r^3$

Volume of hemisphere, $V_2 = \frac{2}{3}\pi r^3$

and volume of cylinder, $V_3 = \pi r^2 h = \pi r^2 \times r = \frac{3}{3}\pi r^3$

$\therefore \quad V_1 : V_2 : V_3 = \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \frac{3}{3}\pi r^3 = 1 : 2 : 3$

i.e., the ratio of their volumes is 1 : 2 : 3.

Hence, the given statement is true.

8. If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of the edge of the cube is 3 cm.

Sol. Let the length of the edge of the cube be a

Then the diagonal of the cube is $\sqrt{3}a$

Also, the length of the diagonal of a cube is $6\sqrt{3}$.

$\therefore \quad \sqrt{3}a = 6\sqrt{3} \Rightarrow a = 6$

So, the length of the edge of the cube is 6 cm.

Hence, the given statement is false.

9. If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be $6 : \pi$.

Sol. Let a be the edge of the cube.

As the sphere is inscribed in a cube, the radius of the sphere is $\frac{a}{2}$.

$$V_1 = \text{Volume of cube} = (\text{edge})^3 = a^3$$

$$V_2 = \text{Volume of sphere} = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{1}{6}\pi a^3$$

$$\therefore \quad V_1 : V_2 = a^3 : \frac{1}{6}\pi a^3 = 1 : \frac{\pi}{6} = 6 : \pi$$

So, the ratio of the volume of the cube to the volume of the sphere will be $6 : \pi$.

Hence, the given statement is true.

10. If the radius of a cylinder is doubled and height is halved, the volume will be doubled.

Sol. Let r be the radius and h the height of the cylinder.

$\therefore$ Original volume $(V_1) = \pi r^2 h$

When radius of cylinder is doubled and height is halved (V_2)

$$= \pi(2r)^2 \times \frac{h}{2} = \pi \times 4r^2 \times \frac{h}{2} = 2\pi r^2 h = 2V_1$$

Hence, the given statement is true.

EXERCISE 13.3

1. Metal spheres each of radius 2 cm are packed into a rectangular box of internal dimensions 16 cm × 8 cm × 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Given your answer to the nearest integer. [Use $\pi = 3.14$]

Sol. Internal volume of a rectangular box = 16 cm × 8 cm × 8 cm = 1024 cm^3

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (2)^3 = \frac{100.48}{3} = 33.49 \text{ cm}^3$$

Volume of 16 such spheres = (33.49 × 16) cm^3 = 535.84 cm^3

When 16 spheres are packed, the box is filled with preservative liquid.

Volume of the preservative liquid = 1024 – 535.84 cm^3 = 488.16 cm^3

2. A storage tank in the form of a cube. When it is full of water, the volume of water is 15.625 m³. If the present depth of water is 1.3 m, find the volume of water already used from the tank.

Sol. Let the edge of the cube be x.

Then, its volume $= x^3$,

When the cube (storage tank) is full of water, the volume of water is 15.625 m³.

So, the volume of cube (storage tank) $= 15.625\ m^3$

$\therefore \quad x^3 = 15.625\ m^3$

$\Rightarrow \quad x = \sqrt[3]{15.625} = \sqrt[3]{(2.5)^3} = 2.5\ m$

$\therefore$ Edge of the cube = 2.5 m

Present depth of water in the tank = 1.3 m

Remaining depth = 2.5 m – 1.3 m = 1.2 m

$\therefore$ Volume of water already used in the tank $= 2.5 \times 2.5 \times 1.2 = 7.5\ m^3$

3. Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely submerged in water.

Sol. Amount of water displaced by a solid spherical ball = Volume of solid spherical ball.

$$\text{Volume of spherical ball} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= \frac{88}{21} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10} = 38.808\ cm^3$$

Hence, the amount of water displaced by solid spherical ball when it is completely immersed in water $= 38.808\ cm^3$

4. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?

Sol. Area of canvas required $= \pi r l$

$$l = \sqrt{r^2 + h^2} = \sqrt{(3.5)^2 + (12)^2}$$

$$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5\ m$$

$\therefore \quad$ Area of canvas required $= \pi r l = \frac{22}{7} \times 12\ m \times 12.5\ m = 471.42\ m^2$

5. Two solid spheres made of the same metal have weights 5920 g and 740 g respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

Sol. Two solid spheres made of the same metal have weights 5920 g and 740 g, respectively.

Mass per unit volume is called the density.

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{5920}{D}}{\frac{740}{D}} \Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{5920}{740}$$

$$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{5920}{740} \Rightarrow \frac{r_1^3}{\left(\frac{5}{2}\right)^3} = \frac{5920}{740} \qquad \left[\because r_2 = \frac{1}{2} \times 5\right]$$

$$\Rightarrow \quad r_1^3 = \frac{5920}{740} \times \frac{125}{8} = 125 \Rightarrow r_1 = (125)^{1/3} = 5 \text{ cm}$$

Hence, the radius of the larger sphere = 5cm

6. A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height of 12 cm, find how many litres of milk is needed to serve 1600 students.

Sol. Radius of cylindrical glass $= 7 \div 2 = 3.5$ cm.

Glass is filled with milk upto an height of 12 cm.

$$= \pi r^2 h = \frac{22}{7} \times (3.5)^2 \times 12$$

$$= \frac{22}{7} \times 12.25 \times 12 = 22 \times 1.75 \times 12 = 462 \text{ cm}^3$$

Quantity of milk needed for 1600 students = (462×1600) cm^3

= 739.2 litres.

7. A cylindrical roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it made?

Sol. Length (height) of cylindrical roller is 2.5 m and radius of the roller is 1.75 m.

In one revolution area covered = lateral surface area of the cylinder

$$2\pi rh = 2 \times \frac{22}{7} \times 1.75 \times 2.5 \text{ m}^2$$

$$44 \times 0.25 \times 2.5 = 27.5 \text{ m}^2$$

Total area on the road covered by cylindrical roller = 5500 m^2.

Hence, number of revolutions made by the roller

$$= \frac{\text{Total area covered}}{\text{Area covered in one revolution}}$$

$$= \frac{5500}{27.5} = 200 \text{ revolutions}$$

8. A small village, having a population of 5,000 requires 75 litres of water per head per day. The village has got an over head tank of measurement 40 m × 25 m × 15 m. For how many days will the water of this tank last?

Sol. Water contained in overhead tank

$$= (40 \times 25 \times 15)\ m^3$$
$$= (40 \times 25 \times 15 \times 1,000) \text{ litres}$$

Water needed for 5,000 villagers for one day

$$= (5,000 \times 75) \text{ litres} = 375000 \text{ litres}$$

∴ Number of days the water of the tank last

$$= \frac{40 \times 25 \times 15 \times 1000}{375000} = \frac{15000}{375} = 40 \text{ days}$$

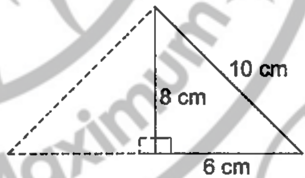
9. A shopkeeper has one spherical ladoo of radius 5 cm. With the same amount of material, how many ladoos of radius 2.5 cm can be made?

Sol. Number of ladoos $= \dfrac{\text{Volume of spherical ladoo of radius 5 cm}}{\text{Volume of one spherical ladoo of radius 2.5 cm}}$

$$= \frac{\frac{4}{3} \times \frac{22}{7} \times (5)^3}{\frac{4}{3} \times \frac{22}{7} \times \left(\frac{5}{2}\right)^3} = 8$$

10. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the sides 8 cm. Find the volume and the curved surface area of the solid so formed.

Sol. When a right triangle is revolved about one of its sides (containing the right angle), a cone is formed.



Now, we have,

Radius (r) of the base of the cone = 6 cm

Height (h) of the cone = 8 cm

and the slant height (l) = 10 cm

$$\text{Now, volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 8$$

$$= \frac{22 \times 12 \times 8}{7} = 301.71 \text{ cm}^3$$

$$\text{Curved surface area of cone} = \pi r l = \frac{22}{7} \times 6 \times 10 = \frac{1320}{7}$$

$$= 188.57 \text{ cm}^2$$

EXERCISE 13.4

1. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cm^3 of iron has been used in making the tube?

Sol. A cylindrical tube is made of iron sheet. which is 2 cm thick.

Its outer radius $= 16 \div 2 = 8$

Thickness of iron sheet $= 2$ cm

and its inner radius $= 8$ cm $- 2$ cm $= 6$ cm.

Height of cylinder $= 100$ cm

Quantity of iron used in making the tube $= \pi(R^2 - r^2)\,h$

$$= \frac{22}{7} \times (8^2 - 6^2) \times 100 = \frac{22}{7}(64 - 36) \times 100$$

$$= \frac{22}{7} \times 28 \times 100 = 8800 \text{ cm}^3$$

2. A semicircular sheet of metal of diameter 28 cm is bent into a form of open conical cup. Find the capacity of the cup.

Sol. Diameter of semicircular sheet is 28 cm. It is bent to form an open conical cup. The radius of sheet becomes the slant height of the cup. The circumference of the sheet becomes the circumference of the base of the cone.

$\therefore$ $l =$ slant height of conical cup $= 14$ cm.

Let r cm be the radius and h cm be the height of the conical cup circumference of conical cup circumference of the semicircular sheet

$\therefore$ $2\pi r = \pi \times 14 \Rightarrow r = 7$ cm

Now, $l^2 = r^2 + h^2 \Rightarrow h = \sqrt{l^2 - r^2}$

$$= \sqrt{(14)^2 - (7)^2} = \sqrt{196 - 49} = \sqrt{147} = 12.12 \text{ cm}$$

$\therefore$ Capacity of the cup

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12.12$$

$$= 622.16 \text{ cm}^3$$

3. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m.

(*i*) How many students can sit in the tent if a student, on a average occupies $\frac{5}{7}$ m^2 on the ground?

(*ii*) Find the volume of the cone.

Sol. (*i*) Number of students who can sit in the tent if a student, on an average, occupies $\frac{5}{7}$ m^2 on the ground

$$= \frac{\pi r^2}{\frac{5}{7}} = \frac{22}{7} \times (5)^2 \times \frac{7}{5} = 110$$

(*ii*) Area of cloth = 165 m^2

Curved surface area of conical tent = $\pi r l$

$\therefore \quad \pi r l = 165$

$$\Rightarrow \quad l = \frac{165}{\pi r} = \frac{165}{\frac{22}{7} \times 5} = \frac{165 \times 7}{22 \times 5} = \frac{21}{2} \text{ m.}$$

Now, $\quad l^2 = r^2 + h^2$

$$\Rightarrow \quad h = \sqrt{l^2 - r^2}$$

$$= \sqrt{\left(\frac{21}{2}\right)^2 - (5)^2}$$

$$\Rightarrow \quad h = \sqrt{\frac{441}{4} - 25} = \frac{\sqrt{341}}{2}$$

$$= \frac{1}{2} \times 18.47 = 9.24 \text{ m (approx.)}$$

Volume of conical tent $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 9.24$$

$$= \frac{550 \times 9.24}{21} = \frac{5082}{21} = 242 \text{ m}^2$$

4. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank fill to its capacity. Calculate the volume of water pumped into the tank.

Sol. Internal diameter of hemispherical tank = 14 m

$\therefore$ Internal radius of hemispherical tank = 14 m ÷ 2 = 7 m

Volume of hemispherical tank $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (7)^3$

$= \frac{44 \times 49}{3} = 718.66 \text{ m}^3.$

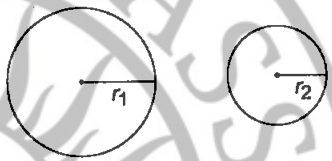
The tank contains 50 kilolitres of water = 50,000 litres = $\frac{50,000}{1,000}$ m^3 = 50 m^3

Volume of water pumped into the tank = 718.66 m^3 – 50 m^3 = 668.66 m^3

5. The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.

Sol. Let V_1 and V_2 be the volumes of two spheres.

$\therefore \quad \frac{V_1}{V_2} = \frac{64}{27}$



$\Rightarrow \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$

$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{64}{27}$

$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \frac{64}{27}$

$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$ ...(1)

Now, let SA_1 and SA_2 be the surface areas of two spheres.

$\therefore \quad \frac{SA_1}{SA_2} = \frac{4\pi r_1^2}{4\pi r_2^2}$

$\Rightarrow \quad \frac{SA_1}{SA_2} = \frac{r_1^2}{r_2^2}$

$\Rightarrow \quad \frac{SA_1}{SA_2} = \left(\frac{r_1}{r_2}\right)^2$

$\Rightarrow \quad \frac{SA_1}{SA_2} = \left(\frac{4}{3}\right)^2$ [Using (1)]

$$\Rightarrow \quad \frac{SA_1}{SA_2} = \frac{16}{9}$$

$\therefore \; SA_1 : SA_2 = 16 : 9$

So, the ratio of the surface areas of the two spheres is 16 : 9.

6. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.

Sol. Side of cube = 4 cm.

Volume of cube $= (4)^3 = 4 \times 4 \times 4 = 64 \text{ cm}^3$

As cube contains a sphere touching its sides, so the diameter of the sphere = 4 cm.

Radius of sphere = 4 cm ÷ 2 = 2 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2)^3$$

$$= \frac{88 \times 8}{21} = \frac{704}{21} = 33.52 \text{ cm}^3$$

Volume of gap in between them $= 64 \text{ cm}^3 - 33.52 \text{ cm}^3 = 30.48 \text{ cm}^3$

7. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceeds its height?

Sol. Volume of sphere $= \frac{4}{3}\pi r^3$

and volume of right circular cylinder $= \pi r^2 h$

$\therefore$ Volume of sphere = Volume of right circular cylinder [Given]

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi r^2 h \Rightarrow h = \frac{4}{3} r$$

Diameter of the cylinder exceeds its height by

$$= 2r - \frac{4}{3} r = \frac{2r}{3}$$

Percentage by which diameter of the cylinder exceeds its height

$$= \frac{\frac{2r}{3}}{\frac{4}{3}r} \times 100 = \frac{2 \times 3}{3 \times 4} \times 100 = 50\%$$

8. 30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid. Find

(*i*) the total surface area.

(*ii*) volume of the cylinder so formed.

Sol. Radius of one circular plate = 14 cm.

Thickness of one circular plate = 3 cm.

As the plates are placed one above the other, so the height of the cylinder formed by placing 30 plates

$$= 30 \times 3 = 90 \text{ cm}$$

(*i*) Total surface area of the cylinder $= 2\pi rh + 2\pi r^2$

$$= 2\pi r(r+h) = 2 \times \frac{22}{7} \times 14(14+90) = 44 \times 208$$

$$= 9152 \text{ cm}^2$$

(*ii*) Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 90 = 55440 \text{ cm}^3$

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