5 Introduction to Euclid's Geometry

Lesson at a Glance

1. The terms point, line and plane are taken as undefined.
2. Axioms (or postulates) are taken as universal truth without proof.
3. Theorems are statements which are proved.
4. **Euclid's Axioms:**
   - Things which coincide with one another are equal to one another.
   - Things which are equal to the same thing are equal to one another.
   - The whole is greater than the part.
   - If equals are added to equals, the wholes are equal.
   - If equals are subtracted from equals, the remainders are equal.
   - Things which are double of the same things are equal to one another.
   - Things which are half of the same things are equal to one another.
5. A point has no part.
6. The ends of a line are points.
7. The edges of a surface are lines.
8. A line has breadthless length.
9. A surface has breadth and length only.
10. **Euclid's Postulates:**
    - Postulate 1: A straight line may be drawn from any one point to any other point.
    - Postulate 2: A terminated line can be produced indefinitely.

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- **Postulate 3:** A circle can be drawn with any centre and any radius.
- **Postulate 4:** All right angles are equal to one another.
- **Postulate 5:** If a straight line falling on two straight lines makes the sum of two interior angles on the same side of it less than two right angles, then the two straight lines meet on that side.

TEXTBOOK QUESTIONS SOLVED

Exercise 5.1 *(Pages – 85-86)*

1. *Which of the following statements are true and which are false? Give reasons for your answers.*

   (*i*) *Only one line can pass through a single point.*

   (*ii*) *There are an infinite number of lines which pass through two distinct points.*

   (*iii*) *A terminated line can be produced indefinitely on both the sides.*

   (*iv*) *If two circles are equal, then their radii are equal.*

   (*v*) *In figure given below, if AB = PQ and PQ = XY, then AB = XY.*

Sol. (*i*) False, through one point, we can draw lines in different directions, so infinite lines can be drawn.

(*ii*) False, only one line passes through two distinct points.

(*iii*) True, according to Euclid's postulate.

(*iv*) True, if we superimpose one circle on the other by coinciding their centres, then their boundrics coincide. So their radii are equal.

(*v*) True, things which are equal to the same things are equal to one another.

2. *Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?*

   (*i*) *Parallel lines* (*ii*) *Perpendicular lines*

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(*iii*) *Line segment* (*iv*) *Radius of a circle* (*v*) *Square*

Sol. (*i*) Lines which do not meet at a point when produced both sides; lines, intersecting lines.

(*ii*) If angle between two given lines is 90°; lines, angle between lines.

(*iii*) A part of a line; two given points, line.

(*iv*) Constant distance from any point on the boundary of a circle to the centre of the circle; centre, boundary.

(*v*) A closed figure of four sides where all the sides are equal and diagonals are equal; points, line segments.

3. *Consider two 'postulates' given below:*

(*i*) *Given any two distinct points A and B, there exists a third point C which is in between A and B.*

(*ii*) *There exist at least three points that are not on the same line.*

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

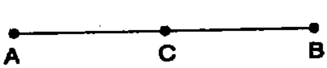
Sol. There are several undefined terms which the student should list. They are consistent, because they deal with two different situations:

(*i*) Says that given two points A and B, there is a point C lying on the line in between them.

(*ii*) Says that given A and B, you can take C not lying on the line through A and B.

These 'postulates' do not follow from Euclid's postulates.

4. *If a point C lies between two points A and B such that AC = BC, then prove that* $AC = \frac{1}{2}AB$. *Explain by drawing the figure.*



Sol. Given: AC = BC

Adding AC to both sides, we get

AC + AC = AC + BC

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$\Rightarrow \quad 2AC = AB \quad \Rightarrow \quad AC = \frac{1}{2}AB.$

5. *In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.*

Sol. Let D is mid-point of AB.

$\therefore \quad AD = BD \quad \Rightarrow \quad AD + AD = AD + BD$

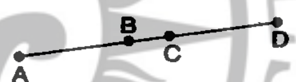
$\Rightarrow \quad 2AD = AB \quad \Rightarrow \quad AD = \frac{1}{2}AB$

From Ans. 4 and 5, we have $AC = AD$

$\Rightarrow$ Points C and D must coincide. Hence mid-point is unique.

6. *In figure given below, if AC = BD, then prove that*

AB = CD.



Sol. Given: $AC = BD$

$\Rightarrow \quad AB + BC = BC + CD$

Subtracting BC from both sides, we get

$AB + BC - BC = BC + CD - BC$

(Subtracting equals from equals)

$\therefore \quad AB = CD.$ **Hence proved.**

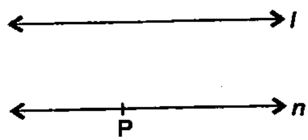
7. *Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate).*

Sol. As statement is true in all the situations. Hence, it is considered a 'universal truth'.

Exercise 5.2 *(Page – 88)*

1. *How would you rewrite Euclid's fifth postulate so that it would be easier to understand?*

Sol. For every line l and for every point P not on l, there is a unique line n through P, which is parallel to l. There can be other statements also.



2. *Does Euclid's fifth postulate imply the existence of parallel lines? Explain.*

Sol. If a straight line l falls on two straight lines m and n such that sum of the interior angles on one side of l is two right angles, then the lines, if produced indefinitely, will not meet on this side of l. In the same way, the sum of the interior angles on the other side of line l will also be two right angles. Therefore, they will not meet on the other side also. So, the lines m and n never meet and are, therefore, parallel.

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