12 Heron's Formula

Lesson at a Glance

1. Area of a triangle = $\frac{1}{2}$ × base × height.
2. Heron derived a formula to obtain the area of a triangle.
3. If the sides of a triangle are a, b and c, then Heron's formula to calculate the area of the triangle is given below:

   Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

   where, $s = \frac{a+b+c}{2}$
4. 's' represents in Heron's formula the semiperimeter of the triangle.
5. Area of a quadrilateral whose all the sides and one diagonal are given, can be calculated by dividing it into triangles and then using Heron's formula.
6. Area of a scalene triangle can be calculated easily by Heron's formula.

TEXTBOOK QUESTIONS SOLVED

Exercise 12.1 *(Pages – 202-203)*

1. *A traffic signal board, indicating 'SCHOOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?*

Sol. Triangle is an equilateral triangle of side a.

$$\therefore \quad s = \frac{a+a+a}{2} = \frac{3a}{2}.$$

Using Heron's formula, we get

Area of the triangle $= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

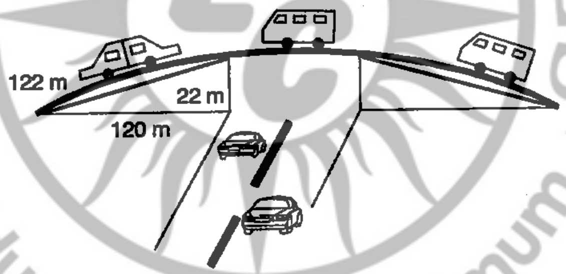
$$= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \frac{\sqrt{3}}{4}a^2$$

If perimeter = 180 cm, then side $= \frac{180}{3} = 60$ cm.

$\therefore$ Area of the triangle

$$= \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3}\ \text{cm}^2.$$

2. *The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see figure). The advertisements yield an earning of ₹ 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?*



Sol. Sides of triangular region for advertisements are 122 m, 120 m, 22 m.

$$\therefore \quad s = \frac{122+120+22}{2} = \frac{264}{2} = 132 \text{ m}.$$

$$\therefore \quad \text{Area} = \sqrt{(132)(132-122)(132-120)(132-22)}\ \text{m}^2$$

$$= \sqrt{132\times10\times12\times110}\ \text{m}^2$$

$$= \sqrt{11\times12\times10\times12\times11\times10}\ \text{m}^2$$

$$= 10 \times 11 \times 12\ \text{m}^2 = 1320\ \text{m}^2.$$

Rent paid for 3 months $= ₹\ 1320 \times 5000 \times \frac{3}{12}$

$= ₹\ 16,50,000.$

3. *There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see figure). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.*

Sol. As the sides of the wall are 15 m, 11 m and 6 m.

$$\therefore \quad s = \frac{15+11+6}{2} = \frac{32}{2} = 16 \text{ m.}$$

$$\therefore \quad \text{Area} = \sqrt{16(16-15)(16-11)(16-6)}$$

$$= \sqrt{16 \times 1 \times 5 \times 10} = \sqrt{800} = 20\sqrt{2} \text{ m}^2.$$

4. *Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.*

Sol. Perimeter = 42 cm. Third side = (42 – 18 – 10) cm = 14 cm.

$$\therefore \quad s = \frac{42}{2} = 21 \text{ cm}$$

$$\therefore \quad \text{Area} = \sqrt{21(21-18)(21-10)(21-14)} \text{ cm}^2$$

$$= \sqrt{21 \times 3 \times 11 \times 7} \text{ cm}^2 = 21\sqrt{11} \text{ cm}^2.$$

5. *Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.*

Sol. Let the sides be $12x$, $17x$ and $25x$.

We have $12x + 17x + 25x = 540$

$\Rightarrow \quad 54x = 540 \quad \Rightarrow \quad x = 10.$

Therefore, sides are 120 cm, 170 cm and 250 cm.

Also, semiperimeter, $s = \dfrac{540}{2} = 270$ cm

Applying Heron's formula, we obtain

Area of the triangle

$$= \sqrt{270(270-120)(270-170)(270-250)}$$

$$= \sqrt{270 \times 150 \times 100 \times 20}$$

$$= \sqrt{9 \times 30 \times 30 \times 5 \times 100 \times 2 \times 10}$$

$$= 3 \times 30 \times 10 \times 10 = 9000 \text{ cm}^2.$$

6. *An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.*

Sol. Each equal side of the isosceles triangle is 12 cm.

$\therefore$ Third side = (30 – 12 – 12) cm = 6 cm.

So, sides of triangle are 12 cm, 12 cm and 6 cm.

$$\therefore \quad s = \frac{12+12+6}{2} = 15$$

Now, area $= \sqrt{15(15-12)(15-12)(15-6)}$

$$= \sqrt{15 \times 3 \times 3 \times 9} = 9\sqrt{15} \text{ cm}^2.$$

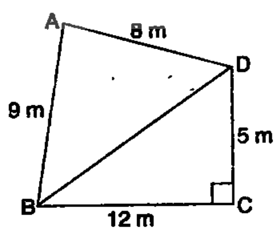
Exercise 12.2 *(Pages – 206-207)*

1. *A park, in the shape of a quadrilateral ABCD, has $\angle C$ = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?*

Sol. Triangle BCD is right angled at C.

$$\therefore \quad BD = \sqrt{(5)^2 + (12)^2} \text{ m} = \sqrt{25+144} \text{ m}$$

$$= \sqrt{169} \text{ m} = 13 \text{ m}.$$



ar(ABCD) = *ar*(ABD) + *ar*(BCD) ...(*i*)

$$ar(BCD) = \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ m}^2 \qquad ...(ii)$$

For triangle ABD,

$$s = \frac{8+9+13}{2} = 15 \text{ m}$$

$$ar(\text{ABD}) = \sqrt{15(15-9)(15-8)(15-13)} \text{ m}^2$$

$$= \sqrt{15 \times 6 \times 7 \times 2} \text{ m}^2 = 6\sqrt{35} \text{ m}^2$$

$$= 35.5 \text{ m}^2 \text{ (approx.)} \quad ...(iii)$$

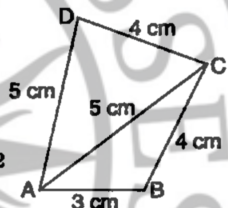
Substituting areas from (*ii*) and (*iii*) in (*i*), we get

$$ar(\text{ABCD}) = 35.5 + 30 = 65.5 \text{ m}^2 \text{ (approx.)}.$$

2. ***Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.***

Sol. $ar(\text{ABCD}) = ar(\text{ABC}) + ar(\text{ACD})$ ...(*i*)

For ΔABC,



$$s = \frac{3+4+5}{2} = 6 \text{ cm}$$

$$\therefore \quad ar(\text{ABC}) = \sqrt{6(6-3)(6-4)(6-5)} \text{ cm}^2$$

$$= \sqrt{6 \times 3 \times 2 \times 1} = 6 \text{ cm}^2 \quad ...(ii)$$

For ΔACD,

$$s = \frac{5+5+4}{2} = 7 \text{ cm}$$

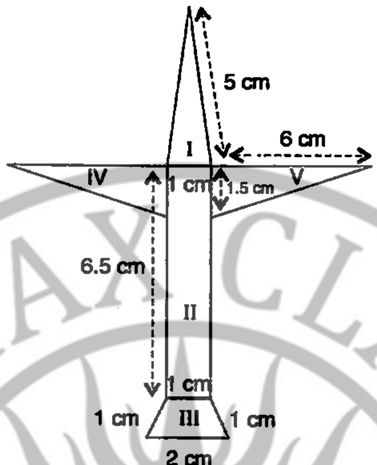
$$\therefore \quad ar(\text{ACD}) = \sqrt{7(7-5)(7-5)(7-4)} \text{ cm}^2$$

$$= \sqrt{7 \times 2 \times 2 \times 3} = 2\sqrt{21} \text{ cm}^2 = 9.17 \text{ cm}^2 \quad ...(iii)$$

Substituting the values of areas from (*ii*) and (*iii*) in (*i*), we get

$$ar(\text{ABCD}) = (6 + 9.17) \text{ cm}^2 = 15.17 \text{ cm}^2.$$

3. ***Radha made a picture of an aeroplane with coloured paper as shown in figure given below. Find the total area of the paper used.***



Sol. Region I: There is a triangle.

Sides of the triangle are 5 cm, 5 cm and 1 cm.

$$s = \frac{5+5+1}{2} = 5.5 \text{ cm}$$

$$\text{Area (I)} = \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)} \text{ cm}^2$$

$$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5} \text{ cm}^2 = \sqrt{6.19} \text{ cm}^2$$

$$= 2.5 \text{ cm}^2 \text{ (approx.)}$$

Region II: There is a rectangle.

Area (II) = 6.5×1 cm^2 = 6.5 cm^2.

Region III: There is a trapezium.

Parallel sides are 2 cm, 1 cm and other sides are 1 cm and 1 cm.

Look the trapezium in the adjacent figure.

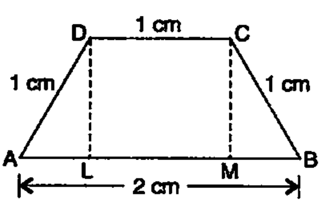
AL = 0.5 cm

$$\text{DL} = \sqrt{(1)^2 - (0.5)^2} \text{ cm}$$

$$= \sqrt{1 - 0.25} \text{ cm}$$

$$= \sqrt{0.75} \text{ cm}$$

$$= 0.9 \text{ cm (approx.).}$$



$\therefore$ Area (III) $= \frac{1}{2}(2 + 1) \times 0.9 = 3 \times 0.45 = 1.35 \text{ cm}^2$.

Regions IV and V: There are two equal right angled triangles.

For each, two perpendicular sides are 6 cm and 1.5 cm.

Area (IV and V) $= 2 \times \frac{1}{2} \times 6 \times 1.5 = 9 \text{ cm}^2$.

$\therefore$ Total area of the paper used

$= \text{area (I + II + III + IV + V)}$

$= (2.5 + 6.5 + 1.35 + 9.0) \text{ cm}^2$

$= 19.35 \text{ cm}^2$.

Hence, the required area is approximately 19.4 cm^2.

4. *A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.*

Sol. Area of parallelogram = area of triangle. ...(*i*)

Let h be the height in cm of the parallelogram.

Sides of the triangle are 26 cm, 28 cm and 30 cm.

$$\therefore \quad s = \frac{26+28+30}{2} = 42.$$

Now, area of the triangle

$$= \sqrt{42(42-26)(42-28)(42-30)} \text{ cm}^2$$

$$= \sqrt{42 \times 16 \times 14 \times 12} \text{ cm}^2$$

$$= 336 \text{ cm}^2$$

Area of the parallelogram = base × height = $28 \times h$

From (*i*), we have

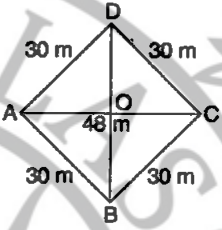
Substituting the values of areas from (*ii*) and (*iii*) in (*i*), we have

$$28 \times h = 336 \Rightarrow h = 12 \text{ cm}.$$

Hence, height of the parallelogram = 12 cm.

5. *A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?*

Sol. In right angled ΔAOD, AD = 30 m, AO = $\frac{48}{2}$ m = 24 m.



$\therefore \quad DO = \sqrt{(30)^2 - (24)^2}$

$m = \sqrt{900 - 576}$ m

$= \sqrt{324}$ m = 18 m.

$\therefore \quad DB = 2 \times 18$ m = 36 m.

$\therefore$ Area of the rhombus (field) = $\frac{1}{2} \times 48 \times 36 = 864$ m^2

Area of grass field for each cow = $\frac{864}{18}$ m^2 = 48 m^2.

6. *An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm, and 50 cm. How much cloth of each colour is required for the umbrella?*

Sol. Each piece is triangular piece
(5 pieces of one colour + 5 pieces of other colour).
There is a total of 10 pieces.
Dimensions of a piece are 50 cm, 50 cm, 20 cm.

$$s = \frac{50 + 50 + 20}{2} = 60 \text{ cm.}$$

$\therefore$ Area of one piece = $\sqrt{60(60-50)(60-50)(60-20)}$ cm^2

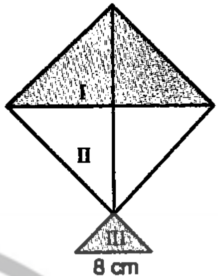
$= \sqrt{60 \times 10 \times 10 \times 40}$

$= 200\sqrt{6}$ cm^2.

$\therefore$ Area of cloth of each colour = $5 \times 200\sqrt{6}$ cm^2

$= 1000\sqrt{6}$ cm^2.

7. *A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?*



Sol. Square have diagonals 32 cm each.

$\therefore$ Area of square $= \frac{1}{2} \times 32 \times 32 \text{ cm}^2 = 512 \text{ cm}^2$.

$\therefore$ Paper used for each of the shades I and II

$$= \frac{512}{2} = 256 \text{ cm}^2.$$

Dimensions of triangle are 8 cm, 6 cm and 6 cm.

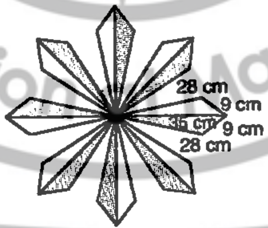
$$\therefore \quad s = \frac{8+6+6}{2} = 10.$$

Area of shade III $= \sqrt{10(10-8)(10-6)(10-6)}$ cm^2

$$= \sqrt{10 \times 2 \times 4 \times 4} = 8\sqrt{5} \text{ cm}^2$$

$$= 17.89 \text{ cm}^2.$$

8. *A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 p per cm^2.*



Sol. Dimensions of one triangular side are 28 cm, 35 cm and 9 cm.

$$\therefore \quad s = \frac{28+35+9}{2} = \frac{72}{2} = 36 \text{ cm}.$$

$\therefore$ Area = $\sqrt{36(36-28)(36-35)(36-9)}$ cm^2

$= \sqrt{36 \times 8 \times 1 \times 27}$ cm^2 = 88.20 cm^2.

$\therefore$ Cost of polishing 16 tiles = ₹ $\frac{50}{100} \times 16 \times 88.20$

= ₹ 705.60.

9. *A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.*

Sol. Draw CE ∥ AD.

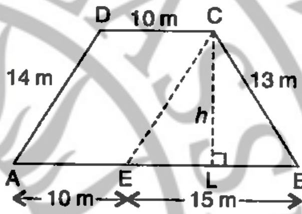
AE = 10 m;

BE = (25 – 10) m = 15 m.

Dimensions of triangle CEB are

CE = 14 m, BE = 15 m,

BC = 13 m.



$\therefore \quad s = \frac{14+15+13}{2} = 21$ m.

Area of triangle BCE = $\sqrt{21(21-14)(21-15)(21-13)}$ m^2

$= \sqrt{21 \times 7 \times 6 \times 8}$ m^2 = 84 m^2.

Let height of the triangle = h m.

$\therefore \quad 84 = \frac{1}{2} \times 15 \times h \Rightarrow h = \frac{84 \times 2}{15}$ m $= \frac{56}{5}$ m.

$\therefore$ Area of the field (trapezium) = $\frac{1}{2}(10 + 25) \times \frac{56}{5}$ m^2

$= \frac{1}{2} \times 35 \times \frac{56}{5}$ m^2 = 196 m^2.