13 Surface Areas and Volumes

Lesson at a Glance

1. Cube, cuboid, cylinder, sphere, cone, hemisphere, frustum of a cone etc. are the solid figures. These are three dimensional figures.
2. Surface is a plane figure.
3. Surface area or total surface area of a cube = $6a^2$, a represents side of a cube.
4. Lateral surface area of a cube = $4a^2$.
5. Perimeter of a cube = $12a$.
6. Volume of a cube = a^3.
7. Length of the diagonal of a cube = $a\sqrt{3}$.
8. Surface area of a cuboid = $2(lb + bh + lh)$, l, b, h are the length, breadth and height of a cuboid respectively.
9. Lateral surface area of a cuboid = $2(l + b) \times h$.
10. Volume of a cuboid = $l \times b \times h$.
11. Length of the diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$.
12. Area of the four walls of a room = $2(l + b) \times h$.
13. The maximum length of a rod that can be put in a cube or a cuboid is equal to the length of its longest diagonal.
14. Curved or lateral surface area of a cylinder = $2\pi rh$, where r and h represent the radius of the base and height of a cylinder respectively.
15. Total surface area of a cylinder = $2\pi r(r + h)$.
16. Volume of a cylinder = $\pi r^2 h$.
17. Surface area of each base of a cylinder = πr^2.
18. Surface area of each base of a hollow cylinder = $\pi(R^2 - r^2)$, where R and r are the base radii of outer and inner cylinder respectively.

19. External curved (lateral) surface area of a hollow cylinder $= 2\pi Rh$.
20. Internal curved (lateral) surface area of hollow cylinder $= 2\pi rh$.
21. Curved surface area of a hollow cylinder $= 2\pi h(R + r)$.
22. Total surface area of a hollow cylinder $= 2\pi(R + r)(h + R - r)$.
23. Volume of a hollow cylinder $= \pi h(R^2 - r^2)$.
24. Curved or lateral surface area of a cone $= \pi rl$, where r and l are the base radius and slant height of a cone respectively.
25. Total surface area of a cone $= \pi r(l + r)$.
26. Volume of a cone $= \frac{1}{3}\pi r^2 h$, where r is base radius and h is height of the cone.
27. Base area of a cone $= \pi r^2$.
28. Volume of a cone $= \frac{1}{3}$(base area) × height.
29. Curved or total surface area of a sphere $= 4\pi r^2$, where r is radius of the sphere.
30. Volume of a sphere $= \frac{4}{3}\pi r^3$.
31. Curved surface area of a hemisphere $= 2\pi r^2$.
32. Total surface area of a hemisphere $= 3\pi r^2$.
33. Volume of a hemisphere $= \frac{2}{3}\pi r^3$.
34. Volume of a spherical shell $= \frac{4}{3}\pi(R^3 - r^3)$, where R and r are radii of outer and inner sphere respectively.

TEXTBOOK QUESTIONS SOLVED

Exercise 13.1 *(Page – 213)*

1. *A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:*

   (i) The area of the sheet required for making the box.

(ii) *The cost of sheet for it, if a sheet measuring 1 m^2 costs ₹ 20.*

Sol. Here, l = 1.5 m, b = 1.25 m, h = 65 cm = 0.65 m.

(i) Area $= 2(l + b) \times h + lb$

$= [2(1.5 + 1.25) \times 0.65 + 1.5 \times 1.25]\ \text{m}^2$

$= (2 \times 2.75 \times 0.65 + 1.5 \times 1.25)\ \text{m}^2$

$= (3.575 + 1.875)\ \text{m}^2 = 5.45\ \text{m}^2$.

(ii) Cost = ₹ 20 × 5.45 = ₹ 109.

2. *The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per m^2.*

Sol. Here, l = 5 m, b = 4 m, h = 3 m.

Area of walls and ceiling $= 2(l + b) \times h + lb$

$= [2(5 + 4) \times 3 + 5 \times 4]\ \text{m}^2$

$= (54 + 20)\ \text{m}^2 = 74\ \text{m}^2$.

Cost = ₹ 7.50 × 74 = ₹ 555.

3. *The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹ 10 per m^2 is ₹ 15000, find the height of the hall.*

[**Hint:** *Area of the four walls = Lateral surface area.*]

Sol. Area of four walls $= 2(l + b) \times h$ = perimeter $\times h = 250 \times h\ \text{m}^2$

Cost of painting = ₹ 10 × 250 × h = ₹ 2500 h

∴ $2500\,h = 15000 \Rightarrow h = 6$ m.

Thus, the height of the hall = 6 m.

4. *The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?*

Sol. Surface area of one brick

$= 2(22.5 \times 10 + 10 \times 7.5 + 22.5 \times 7.5)\ \text{cm}^2$

$= 2(225 + 75 + 168.75)\ \text{cm}^2 = 937.5\ \text{cm}^2$.

∴ Number of bricks which can be painted

$$= \frac{9.375 \times 10000}{937.5} = 100.$$

5. *A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.*

(*i*) *Which box has the greater lateral surface area and by how much?*

(*ii*) *Which box has the smaller total surface area and by how much?*

Sol. For cubical box:

Lateral surface

$= 4(10)^2 \text{ cm}^2$

$= 400 \text{ cm}^2$ ...(*a*)

Total surface area

$= 6(10)^2 \text{ cm}^2$

$= 600 \text{ cm}^2$ ...(*c*)

For cuboidal box:

Lateral surface

$= 2(12.5 + 10) \times 8 \text{ cm}^2$

$= 360 \text{ cm}^2$ ...(*b*)

Total surface area $= 2(12.5 \times 10 + 10 \times 8 + 12.5 \times 8) \text{ cm}^2$

$= 610 \text{ cm}^2$ ...(*d*)

(*i*) Cubical box has greater lateral surface area [From (*a*), (*b*)] by 40 cm^2.

(*ii*) Cubical box has smaller total surface area [From (*c*), (*d*)] by 10 cm^2.

6. *A small indoor greenhouse (herbarium) is made of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.*

(*i*) *What is the area of the glass?*

(*ii*) *How much of tape is needed for all the 12 edges?*

Sol. (*i*) Area of glass (total surface area)

$= 2(30 \times 25 + 25 \times 25 + 30 \times 25) \text{ cm}^2$

$= 2(750 + 625 + 750) \text{ cm}^2 = 4250 \text{ cm}^2$.

(*ii*) Total tap needed $= 4(l + b + h)$

$= 4(30 + 25 + 25) \text{ cm} = 320 \text{ cm}$.

7. *Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹ 4 for 1000 cm², find the cost of cardboard required for supplying 250 boxes of each kind.*

Sol. Area required for 250 bigger boxes

$= 250 \times 2(25 \times 20 + 20 \times 5 + 25 \times 5)\ cm^2$

$= 362500\ cm^2$ ...(*i*)

Area required for 250 smaller boxes

$= 250 \times 2(15 \times 12 + 12 \times 5 + 15 \times 5)\ cm^2$

$= 157500\ cm^2$ ...(*ii*)

∴ Cardboard required $= (362500 + 157500)\ cm^2$

$= 520000\ cm^2$ [From (*i*), (*ii*)]

Total cardboard required with overlaps

$$= \left[520000 + \frac{5}{100} \times 520000\right] cm^2$$

$= 546000\ cm^2$

$$\text{Cost} = ₹\ 4 \times \frac{546000}{1000} = ₹\ 2184.$$

8. *Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?*

Sol. Total area of tarpaulin required $= 2(l + b) \times h + lb$

$= [2(4 + 3) \times 2.5 + 4 \times 3]\ m^2 = 47\ m^2.$

Exercise 13.2 *(Pages – 216-217)*

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. *The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the base of the cylinder.*

Sol. Given: $2\pi rh = 88 \Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88$

$\Rightarrow r = 1$ cm

∴ Diameter of the base = 2 cm.

2. *It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?*

Sol. Base diameter = 140 cm = 1.4 m. Height (h) = 1 m.

$\therefore$ Base radius (r) = 0.7 m

Total surface area (required metal sheet)

$$= 2\pi r(r + h) = 2 \times \frac{22}{7} \times 0.7\ (0.7 + 1)\ \text{m}^2$$

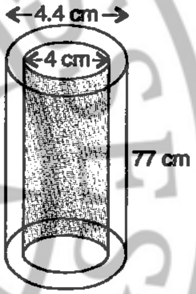
$$= 4.4 \times 1.7\ \text{m}^2 = 7.48\ \text{m}^2.$$

3. *A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see figure). Find its*

(*i*) *inner curved surface area,*

(*ii*) *outer curved surface area,*

(*iii*) *total surface area.*



Sol. Length of the pipe (h) = 77 cm

Inner diameter = 4 cm, inner radius (r_1) = 2 cm, length = 77 cm,

outer diameter = 4.4 cm, outer radius (r_2) = 2.2 cm

(*i*) Inner curved surface area = $2\pi r_1 h$

$$= 2 \times \frac{22}{7} \times 2 \times 77\ \text{cm}^2 = 968\ \text{cm}^2.$$

(*ii*) Outer curved surface area = $2\pi r_2 h$

$$= 2 \times \frac{22}{7} \times 2.2 \times 77\ \text{cm}^2$$

$$= 1064.8\ \text{cm}^2.$$

(*iii*) Total surface area = $2\pi r_1 h + 2\pi r_2 h + 2\pi(r_2^2 - r_1^2)$

$$= 968 + 1064.8 + 2 \times \frac{22}{7}(2.2 + 2)(2.2 - 2)$$

$= 968 + 1064.8 + 5.28$
$= 2038.08 \text{ cm}^2.$

4. ***The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.***

Sol. Area levelled in 1 revolution $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 42 \times 120 \text{ cm}^2$$
$$= 31680 \text{ cm}^2.$$

Area levelled in 500 revolutions (area of playground)

$$= 31680 \times 500 \text{ cm}^2 = \frac{31680 \times 500}{10000} \text{ m}^2$$
$$= 1584 \text{ m}^2.$$

5. ***A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹ 12.50 per m^2.***

Sol. Curved surface of pillar $= 2 \times \frac{22}{7} \times \frac{25}{100} \times 3.5 \text{ m}^2$
$$= 5.5 \text{ m}^2.$$

Cost of painting = ₹ (12.50 × 5.5) = ₹ 68.75.

6. ***Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.***

Sol. $2\pi rh = 4.4 \Rightarrow 2 \times \frac{22}{7} \times 0.7 \times h = 4.4 \Rightarrow h = 1 \text{ m}.$

7. ***The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find***

(i) its inner curved surface area,

(ii) the cost of plastering this curved surface at the rate of ₹ 40 per m^2.

Sol. (*i*) **Inner curved surface area** $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{35}{20} \times 10 \text{ m}^2$$
$$= 110 \text{ m}^2.$$

(ii) Cost of plastering = ₹ (40 × 110) = ₹ 4400.

8. *In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.*

Sol. Total radiating surface $= 2 \times \frac{22}{7} \times \frac{5}{200} \times 28 \text{ m}^2$

$= 4.4 \text{ m}^2$.

9. *Find*

(i) *the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.*

(ii) *how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.*

Sol. (i) Lateral surface area $= 2 \times \frac{22}{7} \times 2.1 \times 4.5 \text{ m}^2$

$= 59.4 \text{ m}^2$.

(ii) Total surface area of the tank $= 59.4 + 2 \times \frac{22}{7} \times 2.1 \times 2.1$

$= 59.4 + 27.72$

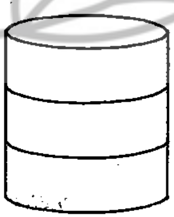
$= 87.12 \text{ m}^2$.

Let total steel used $= x \text{ m}^2$

$\therefore \left(x - \frac{1}{12}x\right) = 87.12 \Rightarrow x = \frac{12}{11} \times 87.12 \text{ m}^2$

$= 95.04 \text{ m}^2$.

10. *In the adjoining figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.*



Sol. Total height including margin = (30 + 2 × 2.5) cm = 35 cm.
Since diameter = 20 cm therefore, radius = 10 cm.

$$\therefore \quad \text{Total cloth required} = 2 \times \frac{22}{7} \times 10 \times 35 \text{ cm}^2$$

$$= 2200 \text{ cm}^2.$$

11. *The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?*

Sol. Total cardboard required

$$= 35 \times \left[2\times\frac{22}{7}\times3\times10.5+\frac{22}{7}\times(3)^2\right] \text{ cm}^2$$

$$= 35(198 + 28.29) \text{ cm}^2 = 7920 \text{ cm}^2.$$

Exercise 13.3 *(Page –221)*

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. *Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.*

Sol. Curved surface area $= \pi rl = \frac{22}{7} \times \frac{10.5}{2} \times 10 \text{ cm}^2$

$$= 165 \text{ cm}^2.$$

2. *Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.*

Sol. Total surface area $= \pi rl + \pi r^2 = \pi r(l + r)$

$$= \frac{22}{7} \times 12 \times (21 + 12) \text{ cm}^2$$

$$= 1244.57 \text{ m}^2.$$

3. *Curved surface area of a cone is 308 cm² and its slant height is 14 cm. Find*

(*i*) *radius of the base and*

(ii) *total surface area of the cone.*

Sol. (i) Given: $\pi rl = 308 \Rightarrow \frac{22}{7} \times r \times 14 = 308$

$$\Rightarrow \quad r = \frac{308}{44} = 7 \text{ cm}$$

$\therefore$ Radius of the base = 7 cm.

(ii) Total surface area = $\pi rl + \pi r^2$

$$= 308 + \frac{22}{7} \times (7)^2 = 462 \text{ cm}^2.$$

4. *A conical tent is 10 m high and the radius of its base is 24 m. Find*

(i) *slant height of the tent.*

(ii) *cost of the canvas required to make the tent, if the cost of 1 m² canvas is ₹ 70.*

Sol. Base radius = 24 m, height = 10 m.

(i) Slant height = $\sqrt{(24)^2 + (10)^2}$ m = $\sqrt{576 + 100}$ m

$$= \sqrt{676} \text{ m} = 26 \text{ m}.$$

(ii) Total canvas required = $\pi rl = \frac{22}{7} \times 24 \times 26 \text{ m}^2$

$$\therefore \quad \text{Cost} = ₹ \left(70 \times \frac{22}{7} \times 24 \times 26\right) = ₹\ 137280.$$

5. *What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use $\pi = 3.14$).*

Sol. Slant height of the tent = $\sqrt{(6)^2 + (8)^2}$ m = $\sqrt{36 + 64}$ m = 10 m.

Area of tarpaulin required = $\pi rl = 3.14 \times 6 \times 10 \text{ m}^2$

$= 188.4 \text{ m}^2$.

$\therefore$ Length of tarpaulin required = $\frac{188.4}{3} = 62.8$ m.

Total length required including wastage = (62.8 + 0.2) m = 63 m.

6. *The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m^2.*

Sol. Curved surface area = $\pi rl = \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 550 \text{ m}^2$.

Cost of white-washing = ₹ $\left(210 \times \frac{550}{100}\right)$ = ₹ 1155.

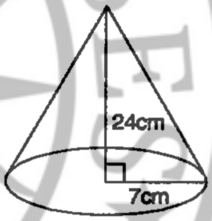
7. *A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.*

Sol. Slant height of joker's cap

$$= \sqrt{(24)^2 + (7)^2} \text{ cm}$$

$$= \sqrt{576 + 49} \text{ cm}$$

$$= \sqrt{625} \text{ cm} = 25 \text{ cm}.$$



∴ Area of sheet required for 10 caps

$$= 10 \times \pi rl = 10 \times \frac{22}{7} \times 7 \times 25 \text{ cm}^2 = 5500 \text{ cm}^2.$$

8. *A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each one has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$.)*

Sol. Slant height of each cone (l) = $\sqrt{(1)^2 + (0.2)^2}$ m

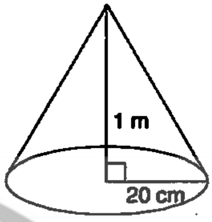
$$= \sqrt{1.04} \text{ m} = 1.02 \text{ m}.$$

∴ Area to be painted for 50 hollow cones

$= 50 \times \pi rl$

$= (50 \times 3.14 \times 0.2 \times 1.02)\ m^2$

$= 32.028\ m^2$



Cost of painting = ₹ (12 × 32.028)

= ₹ 384.34 (approx.).

Exercise 13.4 *(Page –225)*

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. *Find the surface area of a sphere of radius:*

(*i*) *10.5 cm* (*ii*) *5.6 cm* (*iii*) *14 cm.*

Sol. (*i*) Radius of a sphere (r) = 10.5 cm

∴ Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$

$= 1386\ cm^2$

(*ii*) Radius of a sphere (r) = 5.6 cm

∴ Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 5.6 \times 5.6$

$= 394.24\ cm^2$

(*iii*) Radius of a sphere (r) = 14 cm

∴ Surface area of the sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 14 \times 14 = 2464\ cm^2$

2. *Find the surface area of a sphere of diameter:*

(*i*) *14 cm* (*ii*) *21 cm* (*iii*) *3.5 m*

Sol. (*i*) Diameter (d) = 14 cm

∴ Radius (r) = $\frac{14}{2}$ = 7 cm

Now, surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7 = 4 \times 22 \times 7 = 616 \text{ cm}^2.$$

(*ii*) Diameter (d) = 21 cm

$\therefore$ Radius (r) = $\frac{21}{2}$ cm

Now, surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386 \text{ cm}^2$$

(*iii*) Diameter (d) = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2}$ m = $\frac{7}{4}$ m

Now, surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} = \frac{22 \times 7}{4} = 38.5 \text{ m}^2.$$

3. *Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$).*

Sol. Total surface area of hemisphere = $3\pi r^2$

$= 3 \times 3.14 \times 10 \times 10 \text{ cm}^2$

$= 942 \text{ cm}^2$.

4. *The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.*

Sol. S_1 = Surface area with radius 7 cm = $4\pi(7)^2$ cm^2.

S_2 = Surface area with radius 14 cm = $4\pi(14)^2$ cm^2.

$$\therefore \quad \frac{S_1}{S_2} = \frac{4\pi(7)^2}{4\pi(14)^2} = \frac{1}{4} \quad \Rightarrow \quad S_1 : S_2 = 1 : 4.$$

5. *A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs. 16 per 100 cm^2.*

Sol. Inner diameter = 10.5 cm $\Rightarrow$ Inner radius = 5.25 cm.

$$\text{Inner surface area} = 2 \times \frac{22}{7} \times (5.25)^2 \text{ cm}^2 = 173.25 \text{ cm}^2.$$

$$\text{Cost of tin-plating} = ₹\left(16 \times \frac{173.25}{100}\right) = ₹\ 27.72.$$

6. *Find the radius of a sphere whose surface area is 154 cm².*

Sol. Let r be the radius of the sphere.

Surface area of the sphere = 154 cm²

$$4 \times \frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = \frac{154 \times 7}{88} = \frac{49}{4}$$

$$\Rightarrow \qquad r = \frac{7}{2} \text{ cm} \Rightarrow r = 3.5 \text{ cm}.$$

7. *The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.*

Sol. Let diameter of the earth = d units

$\Rightarrow$ Radius of the earth = $\frac{d}{2}$ units

$\therefore$ Diameter of the moon = $\frac{d}{4}$ units

$\Rightarrow$ Radius of the moon = $\frac{d}{8}$ units.

$$\therefore \frac{\text{Surface area of the moon}}{\text{Surface area of the earth}} = \frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} = \frac{4}{64} = \frac{1}{16}$$

$\therefore$ Surface area of the moon : surface area of the earth = 1 : 16.

8. *A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.*

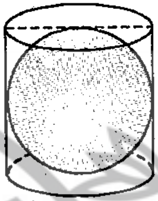
Sol. Inner radius = 5 cm.

$\therefore$ Outer radius = (5 + 0.25) cm = 5.25 cm.

$\therefore$ Outer curved surface area $= 2 \times \frac{22}{7} \times (5.25)^2$ cm^2

$= 173.25$ cm^2.

9. *A right circular cylinder just encloses a sphere of radius r (see figure). Find*



(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

Sol. (i) Surface area of the sphere $= 4\pi r^2$

(ii) For cylinder: radius of base $= r$, height $= 2r$.

$\therefore$ Curved surface area of the cylinder $= 2\pi(r)(2r)$

$= 4\pi r^2$

(iii) Ratio of the required areas is 1 : 1.

Exercise 13.5 *(Page – 228)*

1. *A matachbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?*

Sol. Volume of the packet = 12 × volume of one match box

$= 12 \times 4 \times 2.5 \times 1.5$ cm^3 $= 180$ cm^3.

2. *A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (1 m^3 = 1000 l)*

Sol. Volume of water tank $= 6 \times 5 \times 4.5 = 135$ m^3.

$\therefore$ Water it can hold $= 135 \times 1000\ l = 135000\ l$.

3. *A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?*

Sol. Let height of the vessel be h metres.

Volume of the vessel = 380 m^3

$\Rightarrow \quad 10 \times 8 \times h = 380 \quad \Rightarrow \quad h = 4.75$ m.

4. *Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per m^3.*

Sol. Volume of the cuboidal pit $= 8 \times 6 \times 3$ m^3 $= 144$ m^3.

Cost of digging the pit = ₹ 30 × 144 = ₹ 4,320.

5. *The capacity of a cuboidal tank is 50000 litres of water.*

Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Sol. Let the breadth of the tank be b metres.

Capacity of the tank = 50000 *l*.

$$\text{Volume of the tank} = \frac{50000}{1000}\ \text{m}^3 = 50\ \text{m}^3.$$

$$\therefore \quad 2.5 \times b \times 10 = 50 \quad \Rightarrow \quad b = 2\ \text{m}.$$

6. *A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?*

Sol. Volume of the tank = $20 \times 15 \times 6\ \text{m}^3 = 1800\ \text{m}^3$.

$$\text{Volume of water required per day} = \frac{4000 \times 150}{1000}\ \text{m}^3 = 600\ \text{m}^3.$$

$$\therefore \text{ Number of days water will last} = \frac{1800}{600} = 3 \text{ days.}$$

7. *A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.0 m × 1.25 m × 0.5 m that can be stored in the godown.*

Sol. $$\text{Number of wooden crates} = \frac{40 \times 25 \times 10}{1.0 \times 1.25 \times 0.5} = 16000.$$

Hence, 16000 wooden crates can be stored in the godown.

8. *A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.*

Sol. Volume of the original cube = $(12)^3\ \text{cm}^3 = 1728\ \text{cm}^3$.

$$\text{Volume of one small cube} = \frac{1728}{8}\ \text{cm}^3 = 216\ \text{cm}^3.$$

$$\therefore \quad \text{Side of small cube} = \sqrt[3]{216}\ \text{cm} = 6\ \text{cm}.$$

$$\frac{\text{Surface area of the original cube}}{\text{Surface area of a small cube}} = \frac{6(12)^2}{6(6)^2} = \frac{4}{1}.$$

Hence, the required ratio is 4 : 1.

9. *A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?*

Sol. Rate of water flowing from river is 2 km/hour.

Water flows in 1 minute

$$= \frac{2000}{60} \text{ m} = \frac{100}{3} \text{ m.}$$

$\therefore$ Volume of water flowing into the sea in a minute

$$= 3 \times 40 \times \frac{100}{3} \text{ m}^3 = 4000 \text{ m}^3.$$

Exercise 13.6 *(Pages – 230-231)*

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. *The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? ($1000\ cm^3 = 1\ l$).*

Sol. Circumference of base = 132 cm.

$$\Rightarrow \quad 2\pi r = 132 \quad \Rightarrow \quad r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm.}$$

$\therefore$ Volume of cylindrical vessel = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3 = 34650 \text{ cm}^3.$$

$\therefore$ Capacity of the vessel = $\frac{34650}{1000}\ l = 34.65\ l$

Hence, vessel can hold 34.65 l of water.

2. *The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm³ of wood has a mass of 0.6 g.*

Sol. Volume of the wood = $\pi(r_1^2 - r_2^2)h$

$$= \frac{22}{7} \times \{(14)^2 - (12)^2\} \times 35 \text{ cm}^3$$

$= 110(196 - 144)\text{ cm}^3 = 5720\text{ cm}^3.$

Mass of the pipe (or wood) $= 0.6 \times 5720\text{ g} = 3432\text{ g}$
$= 3.432\text{ kg}.$

3. *A soft drink is available in two packs —*

(i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and

(ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Sol. (*i*) Capacity of tin can = area of base × height
$= (5 \times 4) \times 15\text{ cm}^3 = 300\text{ cm}^3$

(*ii*) Capacity of the plastic cylinder

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10\text{ cm}^3 = 385\text{ cm}^3.$$

Hence, the plastic cylinder has greater capacity by 85 cm^3.

4. *If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find*

(i) radius of its base *(ii) its volume.* *(Use $\pi = 3.14$)*

Sol. Given lateral surface = 94.2 cm^2.

(*i*) $2\pi rh = 94.2 \Rightarrow 2 \times 3.14 \times r \times 5 = 94.2$

$$\Rightarrow r = \frac{94.2}{10 \times 3.14} = 3\text{ cm}.$$

(*ii*) Volume $= \pi r^2 h = 3.14 \times (3)^2 \times 5\text{ cm}^3$
$= 3.14 \times 45\text{ cm}^3 = 141.3\text{ cm}^3.$

5. *It costs Rs. 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹ 20 per m^2, find*

(i) inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

Sol. Cost of the painting is ₹ 2200, if rate is ₹ 20 per m^2.

(*i*) Inner curved surface area $= \dfrac{2200}{20} = 110\text{ m}^2.$

(ii) $2\pi rh = 110 \Rightarrow 2 \times \frac{22}{7} \times r \times 10 = 110$

$\Rightarrow r = \frac{7}{4}$ m = 1.75 m

(iii) Capacity of the vessel $= \pi r^2 h = \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 10$ m^3

$= 96.25$ m^3.

6. *The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?*

Sol. Capacity of closed cylindrical vessel

= 15.4 l

= 15.4 × 1000 cm^3 = 15400 cm^3.

Height = 1 m = 100 cm.

$\therefore \quad \pi r^2 h = 15400 \Rightarrow \frac{22}{7} \times r^2 \times 100 = 15400$

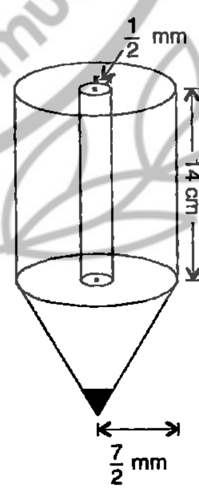
$\Rightarrow \quad r^2 = \frac{154 \times 7}{22} = 49 \Rightarrow r = 7$ cm.

Area of metal sheet needed $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (100 + 7)$ cm^2

$= 4708$ cm^2 = 0.4708 m^2.

7. *A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.*



Sol. Volume of graphite

$= \pi \left(\frac{1}{20}\right)^2 \times 14$ cm^3

$= \frac{22}{7} \times \frac{1}{400} \times 14$ cm^3

$= 0.11$ cm^3

Volume of wood = volume of cylinder – volume of graphite

$$= \left[\frac{22}{7}\times\left(\frac{7}{20}\right)^2\times 14 - 0.11\right] \text{ cm}^3$$

$= (5.39 - 0.11)$ cm^3

$= 5.28$ cm^3.

8. *A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?*

Sol. Volume of soup for one patient $= \frac{22}{7}\times\left(\frac{7}{2}\right)^2\times 4 \text{ cm}^3 = 154 \text{ cm}^3$.

Volume of soup for 250 patients $= 250 \times 154$ cm^3

$= 38500$ cm^3 $= 38.5$ *l*.

Exercise 13.7 *(Page – 233)*

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. *Find the volume of the right circular cone with*

(i) radius 6 cm, height 7 cm *(ii) radius 3.5 cm, height 12 cm.*

Sol. *(i)* Radius $(r) = 6$ cm

Height $(h) = 7$ cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume} = \frac{1}{3}\times\frac{22}{7}\times(6)^2\times 7 \text{ cm}^3 = 264 \text{ cm}^3$$

(ii) Radius $(r) = 3.5$ cm

Height $(h) = 12$ cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h.$$

$$= \frac{1}{3}\times\frac{22}{7}\times(3.5)^2\times 12 \text{ cm}^3 = 154 \text{ cm}^3.$$

2. *Find the capacity in litres of a conical vessel with*
   (i) *radius 7 cm, slant height 25 cm*
   (ii) *height 12 cm, slant height 13 cm.*

Sol. (i) $r = 7$ cm, $l = 25$ cm.

$$\therefore \quad h = \sqrt{l^2 - r^2} = \sqrt{(25)^2 - (7)^2} \text{ cm}$$

$$= \sqrt{625 - 49} \text{ cm} = \sqrt{576} \text{ cm} = 24 \text{ cm.}$$

$$\therefore \quad \text{Capacity} = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ cm}^3$$

$$= 1232 \text{ cm}^3 = 1.232\ l.$$

(ii) $h = 12$ cm, $l = 13$ cm.

$$\therefore \quad r = \sqrt{l^2 - h^2} = \sqrt{(13)^2 - (12)^2} \text{ cm}$$

$$= \sqrt{169 - 144} \text{ cm} = \sqrt{25} \text{ cm} = 5 \text{ cm.}$$

$$\therefore \quad \text{Capacity} = \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 \text{ cm}^3$$

$$= \frac{2200}{7} \text{ cm}^3 = \frac{11}{35} \text{ litre.}$$

3. *The height of a cone is 15 cm. If its volume is 1570 cm³, find the radius of the base. (Use $\pi = 3.14$).*

Sol. Let r be the required radius.

Height (h) = 15 cm

$$\text{Volume} = 1570 \Rightarrow \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$$

$$\Rightarrow \quad r^2 = \frac{1570 \times 3}{3.14 \times 15} = 100 \quad \Rightarrow \quad r = 10 \text{ cm.}$$

4. *If the volume of a right circular cone of height 9 cm is 48π cm³, find the diameter of its base.*

Sol. Height (h) = 9 cm, radius (r) = ?

$$\text{Volume} = 48\pi \text{ cm}^3 \Rightarrow \frac{1}{3} \times \pi \times r^2 \times 9 = 48\pi$$

$\Rightarrow \quad r^2 = 16 \quad \Rightarrow \quad r = 4$ cm.

Diameter $= 2r = 8$ cm.

5. *A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?*

Sol. Diameter $(d) = 3.5$ m

$$\text{Radius } (r) = \frac{\text{diameter}}{2} = \frac{3.5}{2} \text{ m} = \frac{35}{20} \text{ m}$$

Deep, *i.e.*, height $(h) = 12$ m

$$\text{Capacity of the pit} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times 12 \text{ m}^3$$

$= 38.5 \text{ m}^3 = 38.5$ kilolitres.

$[1 \text{ m}^3 = 1 \text{ k}l]$

6. *The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find*

(*i*) *height of the cone*

(*ii*) *slant height of the cone*

(*iii*) *curved surface area of the cone.*

Sol. Diameter = 28 cm, radius $= \frac{28}{2} = 14$ cm

(*i*) Volume of cone $= 9856 \text{ cm}^3$

$$\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$$

$$\Rightarrow \quad h = \frac{4856 \times 3}{44 \times 14} \quad \Rightarrow \quad h = 48$$

$\therefore$ Height (h) of the cone = 48 cm.

(*ii*) Slant height of cone (l)

$$= \sqrt{(14)^2 + (48)^2} \text{ cm}$$

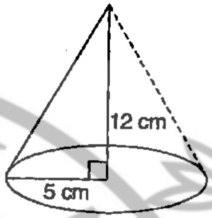
$$= \sqrt{196 + 2304} \text{ cm} = \sqrt{2500} \text{ cm} = 50 \text{ cm.}$$

(*iii*) Curved surface area of the cone $= \pi r l$

$$= \frac{22}{7} \times 14 \times 50 = 2200 \text{ cm}^2.$$

7. *A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.*

Sol. As sides are 5 cm, 12 cm, 13 cm. Hence, triangle is right angled. We have r = 5 cm, h = 12 cm.



$$\therefore \text{ Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 5 \times 5 \times 12 \text{ cm}^3$$

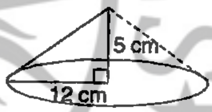
$$= 100\pi \text{ cm}^3.$$

8. *If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.*

Sol. In this case, r = 12 cm, h = 5 cm.

$$\therefore \text{ Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 12 \times 12 \times 5 \text{ cm}^3$$

$$= 240\pi \text{ cm}^3$$



Ratio of volumes obtained in Question 7 and Question 8:

$$\frac{V_1}{V_2} = \frac{100\pi}{240\pi} = \frac{5}{12}, \text{ i.e., } V_1 : V_2 = 5 : 12.$$

9. *A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.*

Sol. Diameter (d) = 10.5 m $\Rightarrow$ radius (r) = $\frac{10.5}{2}$ = 5.25 m

Height (h) = 3 m

Volume of the heap of wheat = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3 \text{ m}^3$$

$$= 86.625 \text{ m}^3.$$

Slant height of the cone = $\sqrt{(5.25)^2 + (3)^2}$ m

= $\sqrt{27.5625 + 9}$ m

= $\sqrt{36.5625}$ m ≈ 6.05 m.

Area of canvas required = $\frac{22}{7} \times 5.25 \times 6.05$ m^2

= 99.825 m^2.

Exercise 13.8 *(Page – 236)*

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

1. *Find the volume of a sphere whose radius is*

(i) 7 cm *(ii) 0.63 m.*

Sol. (*i*) Radius (r) = 7 cm

∴ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (7)^3$ cm^3

= $1437\frac{1}{3}$ cm^3.

(*ii*) Volume = $\frac{4}{3} \times \frac{22}{7} \times (0.63)^3$ cm^3

= 1.05 m^3 (approx.).

2. *Find the amount of water displaced by a solid spherical ball of diameter*

(i) 28 cm *(ii) 0.21 m.*

Sol. Volume of water displaced = Volume of solid spherical ball.

(*i*) Diameter = 28 cm ⇒ Radius = 14 cm.

∴ Volume of water displaced = $\frac{4}{3} \times \frac{22}{7} \times (14)^3$ cm^3

= $11498\frac{2}{3}$ cm^3.

(*ii*) Diameter = 0.21 m $\Rightarrow$ Radius = 0.105 m.

$\therefore$ Volume of water displaced $= \frac{4}{3} \times \frac{22}{7} \times (0.105)^3$ cm^3

$= 0.004851$ m^3.

3. *The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?*

Sol. Diameter of the ball = 4.2 cm.

$\Rightarrow$ Radius of the ball = 2.1 cm.

Volume of the ball $= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 \text{ cm}^3 = 38.808 \text{ cm}^3$.

Mass of the ball = density × volume

$= 8.9 \times 38.808$ g

$= 345.39$ g (approx.)

4. *The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?*

Sol. Let diameter of the earth = d units.

$\Rightarrow$ Radius of the earth $= \frac{d}{2}$ units.

Diameter of the moon $= \frac{d}{4}$ units.

$\Rightarrow$ Radius of the moon $= \frac{d}{8}$ units.

$$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{d}{8}\right)^3} = 64.$$

Volume of the moon $= \frac{1}{64}$ volume of the earth.

Hence, the volume of the moon is $\frac{1}{64}$ of the volume of the earth.

5. *How many litres milk can a hemispherical bowl of diameter 10.5 cm hold?*

Sol. Volume of the hemisphere $= \frac{2}{3} \times \frac{22}{7} \times (5.25)^3 \text{ cm}^3$

$= 303.19 \text{ cm}^3$

So, the capacity of the bowl $= \frac{303.19}{1000}\ l$

$= 0.303\ l$ (approx.)

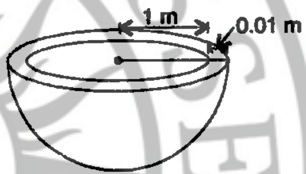
6. *A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.*

Sol. Inner radius = 1 m, thickness

= 1 cm = 0.01 m

Outer radius = 1 + 0.01 m

= 1.01 m



Volume of iron used

$$= \frac{2}{3} \times \frac{22}{7} \times \{(1.01)^3 - (1)^3\} \text{ m}^3$$

$$= \frac{44}{21} \times \{1.0303 - 1\} \text{ m}^3 = \frac{44}{21} \times 0.0303 \text{ m}^3$$

$= 0.06348 \text{ m}^3$ (approx.)

7. *Find the volume of a sphere whose surface area is 154 cm².*

Sol. Surface area $= 154 \text{ cm}^2$

$$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = \frac{154 \times 7}{88} = 12.25$$

$$\Rightarrow \quad r = 3.5 \text{ cm.}$$

Volume of the sphere $= \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 \text{ cm}^3 = 179\frac{2}{3} \text{ cm}^3.$

8. *A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 498.96. If the cost of white-washing is ₹ 2.00 per square metre, find the*

(i) *inside surface area of the dome,*

(ii) *volume of the air inside the dome.*

Sol. Cost of white washing = ₹ 498.96.

Rate of white-washing = ₹ 2 per sq. m.

(i) Inside surface area $= \frac{498.96}{2}$ m^2 = 249.48 m^2.

(ii) We have $2 \times \frac{22}{7} \times r^2 = 249.48$

$$\Rightarrow \quad r^2 = \frac{249.48 \times 7}{44} = 39.69 \quad \Rightarrow \quad r = 6.3 \text{ m}$$

$\therefore$ Volume of the dome $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (6.3)^3 \text{ m}^3$$

$$= 523.90 \text{ m}^3 \text{ (approx.)}.$$

9. *Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the*

(i) *radius r′ of the new sphere,* (ii) *ratio of S and S′.*

Sol. Total volume of 27 sphere $= 27 \times \frac{4}{3}\pi r^3 = 36\pi r^3$

(i) Volume of a new sphere $= \frac{4}{3}\pi r'^3$ ...(i)

$$\therefore \quad \frac{4}{3}\pi r'^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r'^3 = 27r^3 \quad \Rightarrow \quad r' = 3r$$

(ii) Surface area of each of 27 spheres (S) = $4\pi r^2$ ...(a)

Surface area of a new sphere (S′) = $4\pi(3r)^2 = 36\pi r^2$ ...(b)

From (a) and (b), we get

$$\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9}$$

Hence, S : S′ = 1 : 9.

10. *A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?*

Sol. $\because$ Diameter $(d) = 3.5$ mm

$\therefore$ Radius $(r) = \frac{3.5}{2} = 1.75$ mm

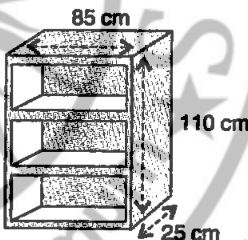
Medicine needed = Volume of capsule $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (1.75)^3 \text{ mm}^3$$

$$= 22.46 \text{ mm}^3 \text{ (approx.)}.$$

Exercise 13.9 *(Pages – 236-237)*

1. *A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.*



Sol. Area of external faces to be polished

= Total surface area of the cuboid

– 3 × area of one rectangular (open) surface

$= [2(85 \times 110 + 110 \times 25 + 85 \times 25) - 3 \times 30 \times 75] \text{ cm}^2$

$= [2(9350 + 2750 + 2125) - 6750] \text{ cm}^2 = 21700 \text{ cm}^2$.

Dimensions of inner boxes are 75 cm × 30 cm × 20 cm

$\therefore$ Area of 3 inner boxes to be painted

$= 3 \times [2(75 + 30) \times 20 + 75 \times 30] \text{ cm}^2$

$= 3[4200 + 2250]\ cm^2$

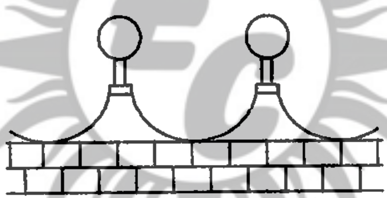
$= 19350\ cm^2$

Total expenses for polishing and painting

$$= ₹\left[\frac{20}{100}\times 21700+\frac{10}{100}\times 19350\right]$$

$= ₹\ (4340 + 1935) = ₹\ 6,275.$

2. *The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the given figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.*



Sol. Surface area of 8 spheres

$$= 8\times\left[4\times\frac{22}{7}\times\left(\frac{21}{2}\right)^2-\frac{22}{7}\times(1.5)^2\right]$$

$= 11031.43\ cm^2.$

Curved surface area of 8 support cylinders

$$= 8\times 2\times\frac{22}{7}\times 1.5\times 7\ cm^2 = 528\ cm^2.$$

$\therefore$ Cost of paint $= ₹\left[\frac{25}{100}\times 11031.43+\frac{5}{100}\times 528\right]$

$= (2757.85 + 26.40) = ₹\ 2784.25.$

3. *The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?*

Sol. Let diameter of the sphere = d units.

$$\Rightarrow \qquad \text{Radius} = \frac{d}{2} \text{ units.}$$

$$\therefore \qquad \text{Surface area} = 4\pi\left(\frac{d}{2}\right)^2 \text{ sq. units} = \pi d^2 \text{ sq. units.} \qquad ...(i)$$

$$\text{Diameter of new sphere} = \left(d - \frac{25}{100}d\right) \text{ units} = \frac{3}{4}d \text{ units.}$$

$$\text{Radius of new sphere} = \frac{3}{8}d \text{ units.}$$

$$\therefore \quad \text{Surface area of new sphere} = 4\pi\left(\frac{3}{8}d\right)^2$$

$$= \frac{9}{16}\pi d^2 \text{ sq. units.} \qquad ...(ii)$$

$$\text{Decrease} = \left(\pi d^2 - \frac{9}{16}\pi d^2\right) \text{ sq. units} = \frac{7}{16}\pi d^2 \text{ sq. units.}$$

$$\therefore \quad \text{Percentage decrease} = \frac{\frac{7}{16}\pi d^2}{\pi d^2} \times 100 = \frac{700}{16}\%$$

$$= 43.75\%.$$

❑❑